

FINAL JEE-MAIN EXAMINATION – FEBRUARY, 2021

(Held On Thursday 25th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

(1) $\frac{1}{27}$ (2) $\frac{3}{4}$ (3) $\frac{1}{8}$ (4) $\frac{3}{8}$

Official Ans. by NTA (3)

2. If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n} \theta$, $y = \sum_{n=0}^{\infty} \sin^{2n} \phi$

and $z = \sum_{n=0}^{\infty} \cos^{2n} \theta \cdot \sin^{2n} \phi$ then :

- (1) $xy - z = (x + y)z$
 (2) $xy + yz + zx = z$
 (3) $xyz = 4$
 (4) $xy + z = (x + y)z$

Official Ans. by NTA (4)

3. Let $f, g : \mathbb{N} \rightarrow \mathbb{N}$ such that $f(n + 1) = f(n) + f(1) \forall n \in \mathbb{N}$ and g be any arbitrary function. Which of the following statements is NOT true?
 (1) If $f \circ g$ is one-one, then g is one-one
 (2) If f is onto, then $f(n) = n \forall n \in \mathbb{N}$
 (3) f is one-one
 (4) If g is onto, then $f \circ g$ is one-one

Official Ans. by NTA (4)

4. The equation of the line through the point $(0,1,2)$ and perpendicular to the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} \text{ is :}$$

(1) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

(2) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

(3) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

(4) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Official Ans. by NTA (4)

5. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4 \alpha + \cos^4 \alpha$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{8}$ (3) $\frac{5}{8}$ (4) $\frac{1}{2}$

Official Ans. by NTA (3)

6. The value of the integral

$$\int \frac{\sin \theta \cdot \sin 2\theta (\sin^6 \theta + \sin^4 \theta + \sin^2 \theta) \sqrt{2 \sin^4 \theta + 3 \sin^2 \theta + 6}}{1 - \cos 2\theta} d\theta \text{ is :}$$

(where c is a constant of integration)

(1) $\frac{1}{18} [11 - 18 \sin^2 \theta + 9 \sin^4 \theta - 2 \sin^6 \theta]^{\frac{3}{2}} + c$

(2) $\frac{1}{18} [9 - 2 \cos^6 \theta - 3 \cos^4 \theta - 6 \cos^2 \theta]^{\frac{3}{2}} + c$

(3) $\frac{1}{18} [9 - 2 \sin^6 \theta - 3 \sin^4 \theta - 6 \sin^2 \theta]^{\frac{3}{2}} + c$

(4) $\frac{1}{18} [11 - 18 \cos^2 \theta + 9 \cos^4 \theta - 2 \cos^6 \theta]^{\frac{3}{2}} + c$

Official Ans. by NTA (4)

7. The value of $\int_{-1}^1 x^2 e^{[x^3]} dx$, where $[t]$ denotes the greatest integer $\leq t$, is :

(1) $\frac{e-1}{3e}$ (2) $\frac{e+1}{3}$ (3) $\frac{e+1}{3e}$ (4) $\frac{1}{3e}$

Official Ans. by NTA (3)

8. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45° . Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

(1) 10 (2) $10\sqrt{3}$

(3) $10(\sqrt{3}+1)$ (4) $10(\sqrt{3}-1)$

Official Ans. by NTA (3)

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it?

- (1) $(-6, 0)$ (2) $(4, 5)$ (3) $(5, 4)$ (4) $(0, 3)$

Official Ans. by NTA (3)

10. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Official Ans. by NTA (4)

11. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is:

- (1) $\frac{3}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $3\sqrt{2}$ (4) $\frac{3}{2\sqrt{2}}$

Official Ans. by NTA (4)

12. The image of the point $(3, 5)$ in the line $x - y + 1 = 0$, lies on :

- (1) $(x - 2)^2 + (y - 2)^2 = 12$
 (2) $(x - 4)^2 + (y + 2)^2 = 16$
 (3) $(x - 4)^2 + (y - 4)^2 = 8$
 (4) $(x - 2)^2 + (y - 4)^2 = 4$

Official Ans. by NTA (4)

13. If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$

intersect each other at an angle of 90° , then which of the following relations is TRUE?

- (1) $a + b = c + d$
 (2) $a - b = c - d$
 (3) $a - c = b + d$
 (4) $ab = \frac{c+d}{a+b}$

Official Ans. by NTA (2)

14. $\lim_{n \rightarrow \infty} \left(1 + \frac{1 + \frac{1}{2} + \dots + \frac{1}{n}}{n^2} \right)^n$ is equal to :

- (1) $\frac{1}{2}$ (2) 0 (3) $\frac{1}{e}$ (4) 1

Official Ans. by NTA (4)

15. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:

- (1) $\frac{1}{72}$ (2) $\frac{5}{216}$ (3) $\frac{1}{36}$ (4) $\frac{1}{54}$

Official Ans. by NTA (2)

16. The total number of positive integral solutions (x, y, z) such that $xyz = 24$ is :

- (1) 36 (2) 24 (3) 45 (4) 30

Official Ans. by NTA (4)

17. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in \mathbb{R} , is :

- (1) 3 (2) 2 (3) 0 (4) 4

Official Ans. by NTA (1)

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is

$$\frac{x^2 - 4x + y + 8}{x - 2}$$

through the point:

- (1) $(5, 4)$ (2) $(4, 5)$ (3) $(4, 4)$ (4) $(5, 5)$

Official Ans. by NTA (4)

19. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to :

- (1) $A \rightarrow (A \wedge B)$ (2) $A \rightarrow (A \rightarrow B)$
 (3) $A \rightarrow (A \leftrightarrow B)$ (4) $A \rightarrow (A \vee B)$

Official Ans. by NTA (4)

20. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with

$$f'\left(\frac{4}{3}\right) = 0$$

- (1) $(5, 8)$ (2) $(-5, 8)$
 (3) $(5, -8)$ (4) $(-5, -8)$

Official Ans. by NTA (1)

SECTION-B

1. Let $f(x)$ be a polynomial of degree 6 in x , in which the coefficient of x^6 is unity and it has

extrema at $x = -1$ and $x = 1$. If $\lim_{x \rightarrow 0} \frac{f(x)}{x^3} = 1$, then

$5 \cdot f(2)$ is equal to _____.

Official Ans. by NTA (144)

2. The number of points, at which the function $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in \mathbb{R}$ is not differentiable, is _____.

Official Ans. by NTA (2)

3. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A . Then A^4 is equal to _____.

Official Ans. by NTA (64)

4. Let A_1, A_2, A_3, \dots be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1} . If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is _____.

Official Ans. by NTA (9)

5. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real

numbers such that $x + y + z > 0$ and $xyz = 2$.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _____.

Official Ans. by NTA (7)

6. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and

$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _____.

Official Ans. by NTA (13)

7. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _____.

Official Ans. by NTA (32)

8. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If \vec{r} is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to _____.

Official Ans. by NTA (12)

9. If the system of equations

$$kx + y + 2z = 1$$

$$3x - y - 2z = 2$$

$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then k is equal to _____.

Official Ans. by NTA (21)

10. The locus of the point of intersection of the lines

$$(\sqrt{3})kx + ky - 4\sqrt{3} = 0 \quad \text{and}$$

$$\sqrt{3}x - y - 4(\sqrt{3})k = 0$$

is a conic, whose eccentricity is _____.

Official Ans. by NTA (2)