JEE Main Feb 2021

Memory based questions and Solutions

24 Feb 2021 - Evening Shift

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Physics

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Q-1)

A wire of 2 Ω has and length of 1 m It is stretched till its length increases by 25 % The percentage change in resistance to the nearest integer is
1) 25% 2) 76% 3) 125 % 4) 56%

Ans-

4) 56%

Solution-

$$R = 2\Omega$$

$$l_1 = 1m$$
$$l_2 = 1.25m$$

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{Al}$$

$$\frac{R_1}{R_2} = (\frac{l_1}{l_2})^2 = (\frac{1}{\frac{25}{16}}) = \frac{16}{25}$$

$$R_2 = \frac{25}{16}R_1$$

$$\frac{\Delta R}{R_1} = \frac{R_2 - R_1}{R_1} * 100 = \frac{9}{16} * 100 = 56\%$$

Q-2)

A particle excutes S.H.M. the graph of velocity as a displacement is :
1) a circle 2) an ellipse 3) a helix   4) a parabola

Ans-

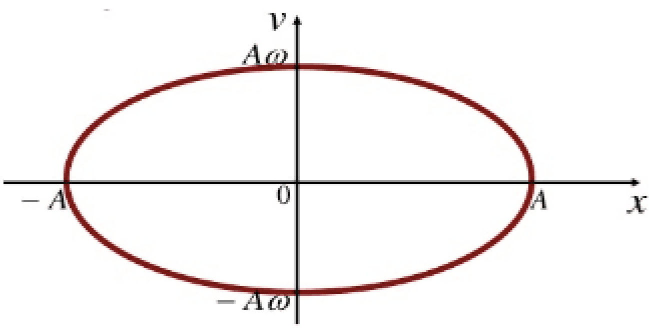
2) an ellipse

Solution-

$$v = \omega\sqrt{A^2 - x^2}$$
$$v^2 = \omega^2 A^2 - \omega^2 x^2$$
$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

$$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$

Graph of v vs x is ellipse



Q-3)

The recoil speed of a hydrogen atom after it emits a photon in going from n=5 state to n=1 state is

1) 3.25 m/s
2) 4.17 m/s
3) 4.34 m/s
4) 2.19 m/s

Ans-

2) 4.17 m/s

Solution-

$$\text{Momentum, } \mathbf{p} = \frac{\mathbf{E}}{\mathbf{c}} = \mathbf{mv}$$
$$\text{velocity, } = \mathbf{v} = \frac{\Delta\mathbf{E}}{\mathbf{mc}},$$
$$\text{mass of hydrogen atom } = \mathbf{m} = \mathbf{1.67 \times 10^{-27} Kg}$$
$$\mathbf{E} = \frac{-13.6\mathbf{Z}^2}{\mathbf{n}^2}$$
$$\Delta\mathbf{E} = \mathbf{E}_5 - \mathbf{E}_1 = -13.056\text{eV}$$
$$\text{Putting the value in velocity expression we get } \mathbf{v} = \mathbf{4.175\ ms^{-1}}$$

Q-4.

$\text{If } \lambda = \frac{C}{V}, \text{find the dimension of } \lambda$
.

A) $[M^{-2}L^{-4}T^{7}A^{3}]$

B) $[M^{-4}L^{-3}T^{7}A^{3}]$

C) $[M^{-4}L^{-3}T^{3}A^{4}]$

D) $[M^{-1}L^{-5}T^{3}A^{4}]$

Ans

A) $[M^{-2}L^{-4}T^{7}A^{3}]$

Solution-

$$\frac{C}{V} = \lambda$$
$$\left[\frac{C}{V}\right] = \left[\frac{Q}{V^2}\right] = \left[\frac{AT}{\frac{M^2L^4T^{-4}}{A^2T^2}}\right] = [M^{-2}L^{-4}T^{7}A^{3}]$$

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Chemistry

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2. 2,4 DNP test can be used to identify

1. ether 2. aldehyde 3. halogens 4. amine

Solution:

2,4 DNP test is used to identify aldehydes and Ketones

Hence, Option 2 is correct.

7. Which pair of oxides is acidic in nature?

1. N_2O, BaO 2. B_2O_3, SO_2 3. CaO, SiO_2 4. Be_2O_3, CaO

Solution:

Correct option 2) B_2O_3, SiO_3 are acidic and they form boric acid H_3BO_3 and silicic acid H_4SiO_4 on reaction with water.

10. The correct order of electron gain enthalpy is

1. O>S>Se>Te 2. S>Se>Te >O 3. Te> Se> S>O 4. S>O>Se>Te

Solution:

The correct order of electron gain enthalpy is

$S > Se > Te > O$

Due to the small size of Oxygen, it has the least negative value of electron gain enthalpy

Hence, option 2 is correct.

13. In $CH_2 = C = CH - CH_3$ molecule the hybridization of carbon 1,2,3 and 4 respective are

1. sp^2, sp, sp^2, sp^3 2. sp^2, sp^2 , sp^2, sp^3 3.sp^3, sp, sp^3, sp^3 4. sp^2, sp^3, sp^2, sp^3

Solution:

Hybridisation of carbons

1 - sp^2

2 - sp

3 - sp^2

4 - sp^3

Correct option is 1) sp^2 ,sp ,sp^2 ,sp^3

17. Which of the following forms of hydrogen emits low energy of particles?

1. Tritium $\frac{3}{1}H$ 2. Proton H 3. protium $\frac{1}{1}H$ 4. Deuterium

Solution:

The correct option is 1) Tritium ${}^3_1H$

Tritium is the only radioactive that emits low energy β^- particles.

20. Seliwanoff test and Xantha protein test are used of the indentification - and - respectively

1) aldoses, ketoses

2) ketoses, aldoses

3) Proteins, ketoses

4) Ketoses, Proteins

Solution:

Seliwanoff's test is used to identity ketoses.

Xanthoproteic test is used to identify proteins.

Hence option d) ketoses, proteins is correct.

21) The average S-F bond energy in kJ mol $^{-1}$ of SF_6 is _ rounded off to the nearest integer (given) the values of standard enthalpy of formation of $SF_6\,(g)$, $S\,(g)$ and $F(g)$ are -1100, 275, 80 kJmol^{-1}:

Solution:

$$\frac{1}{8}S_8(s) + 3F_2(g) \rightarrow SF_6(g)$$
$$\frac{1}{8}S_8(s) \rightarrow S(g) \quad \Delta H = 275KJmol^{-1}$$
$$3F_2(g) \rightarrow 6F(g) \quad \Delta H = 80KJmol^{-1}$$
$$S(g) + 6F(g) \rightarrow SF_6(g)\Delta H = xKJmol^{-1}$$

According to Bond energy concept,

$$\Delta H_f^0(S) + 6\Delta H_f^0(F) - 6\Delta H_{BE}^0(SF) = \Delta H_f^0(SF_6)$$

$$275 + 6 \times 80 - 6x = -1100$$
$$6x = 275 + 480 + 1100$$
$$6x = 1855$$
$$x = 309.17KJmol^{-1}$$

22) If the activation energy of a reaction is 80.9 kJ mol $^{-1}$, the fraction of molecules at 700 k having enough energy to reach to form products is e^{-x}.The value of x is __ [use R = 8.31 J.K^{-1} mol $^{-1}$]

Solution:

Given,

$$E_a = 80.4KJmol^{-1}$$
$$R = 8.31$$
$$T = 700K$$

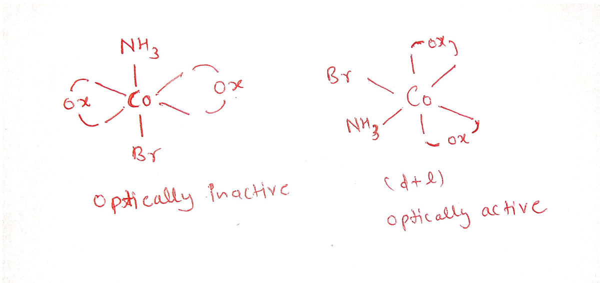
$\therefore$ The fraction of molecules having energy greater than activation energy = $e^{\frac{-E_a}{RT}}$

$$\therefore x = \frac{E_a}{RT} = \frac{80.4 \times 10^3}{8.31 \times 700} = 13.82$$

23) The number of stereoisomers possible for $[Co\,(ox)_2\,(Br)\,(NH_3)]$ is __[ox = oxlate]

Solution:

$[\mathrm{Co(ox)_2Br(NH_3)}]$ is an octahedral compound it has 3 stereoisomers.



24) The PH of ammonium Phosphate solution if PK_a of phosphoric acid and PK_b of ammonium hydroxide are 5.23 and 4.75 respectively is __

Solution:

pH of a salt of a weak acid and weak base is given by

$$pH = 7 + \frac{1}{2}(pK_a - pK_b)$$

According to given data,

$$pH = 7 + \frac{1}{2}(5.23 - 4.75) = 7.24$$

25. A ball weighing 10g is moving witha a velocity of a uncertainty in its velocity is 5%. Then the uncertainty in its __ $\times 10^{-33}$ (Given : $h = 6.63 \times 10^{-34}$ Js)

Solution:

Given,

$$\mathrm{m = 10\ g = 10^{-2}\ kg}$$

$$\mathrm{v = 90 ms^{-1}}$$

$$\Delta \mathrm{v} = \frac{5}{100} \times 90 = 4.5\ \mathrm{ms^{-1}}$$

$$\Delta \mathrm{P} = 4.5 \times 10^{-2}$$

Now, according to Heisenberg's Uncertainty Principle

$$(\Delta \mathrm{x}) \times (\Delta \mathrm{P}) \geq \frac{\mathrm{h}}{4\pi}$$

$$(\Delta \mathrm{x}) \geq \frac{\mathrm{h}}{4\pi \times (\Delta \mathrm{P})}$$

$$(\Delta x) \geq \frac{6.63 \times 10^{-34}}{4 \times (3.14) \times (4.5 \times 10^{-34})} > 1.2 \times 10^{-34} \text{ m}$$

Hence, the correct answer is 1.2

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Maths

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1. The triangle of maximum area that can be inscribed in a given circle of radius r is:

  1. An equivalent triangle of height $\frac{2r}{3}$
  2. An equilateral traingle having each of its side of length $\sqrt{3}r$
  3. An isosceles triangle with base equal to 2r
  4. A right angle traigle having two of its sides of length 2r and r.

2. Let abe an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval (a, a+1). Then , $|a|$ is equal to _

Solution:

Given polynomial equation has only one real root lying between (-2,-1)

Hence, a = -2

|a|= 2

3. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)n-1dx$, for m,n >1 and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)m+n}dx = aI_{m,n}a\epsilon R$, then a equals__-

Solution

$$\text{Given } I(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$$

$$\text{Now} \quad \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = aI(m,n)$$

$$\Rightarrow I = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}}dx + \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}}dx$$

$$\Rightarrow I = I_1 + I_2$$

$$I_2 = \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}} dx$$

$$put \quad x = \frac{1}{z}$$

$$I_2 = \int_1^\infty \frac{z^{m-1}}{(1+z)^{m+n}} dz == \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

From Above

$$I = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx + \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

$$= \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+h}} dx = I(m,n)$$

a = 1

6. For x > 0, if $f(x) = \int_1^x \frac{\log e}{1+t} dt$, then $F(e) + F\left(\frac{1^1}{e}\right)$ is equal to

1. 1
2. -1
3. $\frac{1}{2}$
4. 0

Solution

$$f(x) = \int_1^x \frac{\ln t}{1+t} dt$$

$$\text{Now, } f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{\ln t}{1+t} dt$$

$$\text{Let } t = \frac{1}{y} \Rightarrow dt = -\frac{1}{y^2} dy$$

$$= \int_1^y \frac{\ln\left(\frac{1}{y}\right)}{1+\left(\frac{1}{y}\right)} \times -\frac{dy}{y^2}$$
$$= \int_1^y \frac{-\ln(y)}{1+y} \times -\frac{dy}{y}$$
$$= \int_1^y \frac{\ln(y)}{y(1+y)} dy$$

$$= \int_1^t \frac{\ln(t)}{t(1+t)} dt$$

$$f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln(t)}{(1+t)} dt + \int_1^x \frac{\ln(t)}{t(1+t)} dt$$
$$= \int_1^x \frac{(1+t)\ln(t)}{t(1+t)} dt$$
$$= \int_1^x \frac{\ln(t)}{t} dt$$
$$= \left.\frac{(\ln t)^2}{2}\right|_1^x = \frac{(\ln x)^2}{2}$$
$$\text{Thus, } f(e) + f\left(\frac{1}{e}\right) = \left(\frac{\ln e}{2}\right)^2 = \frac{1}{2}$$

7. The triangle of maximum area that can be inscribed in a given circle of radius r is:

1. An equivalent triangle of height $\frac{2r}{3}$
2. An equilateral traingle having each of its side of length $\sqrt{3}r$
3. An isosceles triangle with base equal to 2r
4. A right angle traigle having two of its sides of length 2r and r.

Solution

Let the lowest point on the circle, with radius R,R, lie on the origin of the Cartesian coordinate axes.

let us assume that the base AB of △ ABC is parallel to the X axis. Let the coordinates of points AA and BB be (−x,y)(−x,y) and (x,y)(x,y) respectively.

In △A B C, point C must lie on the Y axis and hence, the coordinates of point C are (0,2 R).
The measures of the base and height of △ A B C are 2x and 2 R-yespectively.

The area of △A B C=x(2 R-y). The equation of the above circle is $x^2 + (y-R)^2 = R^2$

$$x^2 = R^2 - (y^2 - 2yR + R^2) = 2yR - y^2$$
$$\Delta = \sqrt{2yR - y^2}(2R - y)$$

$$\frac{d\Delta}{dy} = \frac{(R-y)}{\sqrt{2yR-y^2}}(2R-y) - \sqrt{2yR-y^2} = 0$$
$$\frac{2R^2 - 3yR + y^2 - 2yR + y^2}{\sqrt{2yR-y^2}} = 0$$
$$\frac{2R^2 - 5yR + 2y^3}{\sqrt{2yR-y^2}} = 0$$
$$\frac{(2R-y)(R-2y)}{\sqrt{y}\sqrt{2R-y}} = 0$$
$$\frac{(R-2y)\sqrt{2R-y}}{\sqrt{y}} = 0$$
$$y = 2R \quad \text{or } y = \frac{R}{2}$$

$$y = \frac{R}{2}, \quad x = \sqrt{2yR-y^2} = \frac{\sqrt{3}}{2}R$$
$$AB = 2x = \sqrt{3}R$$
$$AC = BC = \sqrt{\left(\frac{\sqrt{3}}{2}R - 0\right)^2 + \left(2R - \frac{R}{2}\right)^2} = \sqrt{3}R$$

1 Let a and b be positive two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$, Let $P_n = (\alpha)^n + (\beta)^n Pn - 1 = 11$ and $Pn + 1 = 29$ for some interger n>1. Then the value of Pn^2 is _

Solution

Given $\alpha + \beta = 1 \quad and \quad \alpha\,\beta = -1, P_{n-1} = 11, P_n = 29$

$$AM \quad of \quad P_{n-1} and, P_n = \frac{\alpha^{n-1} + \beta^{n-1} + \alpha^n \beta^n}{2}$$
$$= \frac{\alpha^{n-1}(\alpha+1) + \beta^{n-1}(\beta+1)}{2}$$
$$= \frac{\alpha^{n-1}(\alpha^2) + \beta^{n-1}(\beta^2)}{2}$$
$$\because \alpha^2 = \alpha + 1 \; and \; \beta^2 = \beta + 1$$
$$AM = \frac{1}{2}\left(\alpha^{n+1} + \beta^{n+1}\right) = \frac{1}{2}P_{n+1}$$

$$\frac{P_{n-1} + P_n}{2} = \frac{1}{2}P_{n+1}$$

$$\frac{11 + P_n}{2} = \frac{1}{2} \times 29 \Rightarrow P_n = 29 - 11$$

$$= 18$$

Hence

$(P_n)^2 = (18^2)$

$= 324$ **Ans**

4. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satifies the equation $A^{20} + aA^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers a and β, then β -a is equal to _

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 114 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 108 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^{20} + aA^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$1+a+\beta=1$

$a=-\beta$

$114+108a+2\beta=4$

5. Let L be a common tangent line to the curves $4x^2-+9y^2=36$ and $(2x)^2+(2y)^2=31$. Then the square of the slope of the line L is___

Solution

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

$$x^2+y^2=\frac{31}{4}$$

Tangent of elipse

$$y=mx\pm a\sqrt{a^2m^2+b^2}$$

$$y=mx\pm a\sqrt{9m^2+4}$$

Tangent of circle

$$y=mx\pm\sqrt{\frac{31}{4}}\sqrt{1+m^2}$$

Since tangent is common

$$\sqrt{9m^2+4}=\sqrt{\frac{31}{4}}\sqrt{1+m^2}$$

$$36m^2+16=31+31m^2$$

$$m^2=3$$

|

1) Let $f\left(x\right)=\int_0^t e^t dt+e^x$ be a differentiable for all $x\ \epsilon\ R$. Then f(x) equals.

1) $e\left(e^x-1\right)$

2) $\left(2e^x-1\right)$

3) $2e\left(e^x-1\right)-1$

4) e^x-1

Solution

$$f(x)=\int_0^x e^t dt+e^x$$

$$f(x)=[e^t]_0^x+e^x$$

$$f(x)=2e^x-1$$

2) A natural number has Prime factorization given by $n=2^x3^y5^z$ where y and z are such that $y+z=5$ and $y^{-1}+z^{-1}=\frac{5}{6}, y>z.$ Then the number of odd divisors of n including 1, is :

1) 11

2) 6x

3) 12

4) 6

Solution

$$n=2^x3^y5^z$$

$$y+z=5$$

$$\frac{1}{y}+\frac{1}{z}=\frac{5}{6}$$

y = 3

z =2

Answer : 1x4x3 = 12

3) A seven digit number is formed using digits $3,3,4,4,5,5.$ Then probility, that number so formed is divisible by 2, is :

1) $\frac{1}{7}$

2) $\frac{3}{7}$

3) $\frac{4}{7}$

4) $\frac{6}{7}$

Solution

$$P(E) = \frac{7!}{3!2!2!}$$

$$P(A) = \frac{6!}{2!2!2!}$$

$$\frac{P(A)}{P(E)} = \frac{3}{7}$$

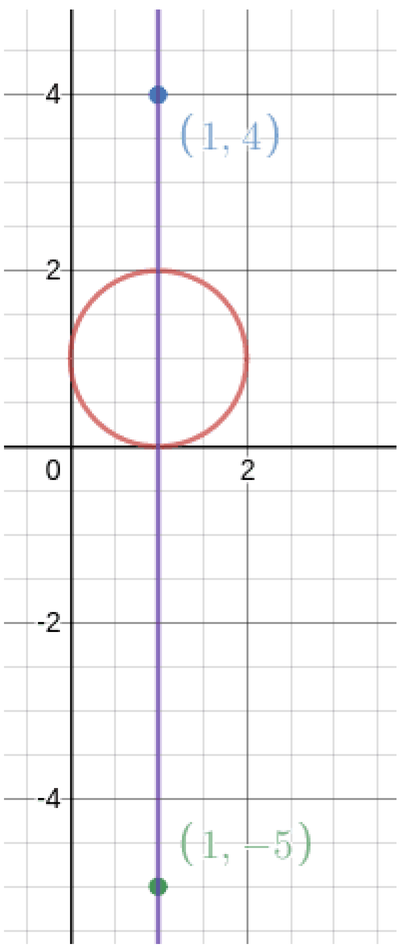
8) Let $A\ (1, 4)$ and $B\ (1, -5)$ be two points. Let P point on the circle $(x-1)^2 + (y-1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value , then the points P,A and B lie on :

1) an ellipse

2) a parabola

3) a hyperbola

4) a straight line



Clearly it is straight line.

10) Let $f\ (x)$ be a differentiable function at $x = a$ with $f\ (a)$ and $f\ (a) = 4$ then $\lim_{x \to a} \frac{xf\ (a) - af\ (x)}{x - a}$

1) $2a-4$

2) $2a+4$

3) $a+4$

4) $4-2a$

Solution

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$$

at x=a

$$\text{L-Hospital}$$

$$=\frac{f(a)-af'(a)}{1}$$

$$=4+2a$$

20) If the focus of the mid - point of the line segment from the point $(3,2)$ to a point on the circle, $x^2+y^2=1$ is a circle of radius r then r is equal to

1) $\frac{1}{3}$

2) $\frac{1}{2}$

3) $\frac{1}{4}$

4) 1

$$General\ point\ on\ the\ circle$$
$$(\cos\theta,\sin\theta)$$