Q-1)
A wire of 2 Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in resistance to the nearest integer is:
1) 25%, 2) 76%, 3) 125%, 4) 56%

Ans-
4) 56%

Solution-

\[ R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} \]

\[ \frac{R_1}{R_2} = \left( \frac{l_1}{l_2} \right)^2 = \left( \frac{1}{2.25} \right) = \frac{16}{25} \]

\[ R_2 = \frac{25}{16} R_1 \]

\[ \Delta R = \frac{R_2 - R_1}{R_1} \times 100 = \frac{9}{16} \times 100 = 56\% \]

Q-2)
A particle executes SHM. the graph of velocity as a displacement is:
1) a circle 2) an ellipse 3) a helix 4) a parabola

Ans-
2) an ellipse

Solution-

\[ v = \omega \sqrt{A^2 - x^2} \]

\[ v_n^2 = \omega^2 A^2 - \omega^2 x^2 \]

\[ v_n^2 + \omega^2 x^2 = \omega^2 A^2 \]

\[ \frac{v_n^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1 \]

Graph of \( v \) vs \( x \) is ellipse

Q-3)

The recoil speed of a hydrogen atom after it emits a photon in going from \( n=5 \) state to \( n=1 \) state is

1) 3.25 m/s
2) 4.17 m/s
3) 4.34 m/s
4) 2.19 m/s

Ans-

2) 4.17 m/s

Solution-

Momentum, \( p = \frac{E}{c} = mv \)

velocity, \( v = \frac{\Delta E}{mc} \),

mass of hydrogen atom \( m = 1.67 \times 10^{-27} \text{ Kg} \)

\[ E = \frac{-13.6Z^2}{n^2} \]

\[ \Delta E = E_5 - E_1 = -13.056 \text{ eV} \]

Putting the value in velocity expression we get \( v = 4.175 \text{ m/s} \)

Q-4.
If $\lambda = \frac{C}{V}$, find the dimension of $\lambda$

\begin{align*}
A) & [M^{-2}L^{-4}T^{-7}A^3] \\
B) & [M^{-4}L^{-3}T^{-7}A^3] \\
C) & [M^{-4}L^{-3}T^{-3}A^4] \\
D) & [M^{-1}L^{-5}T^{-3}A^4]
\end{align*}

Ans

A) $[M^{-2}L^{-4}T^{-7}A^3]$ 

Solution-

\[ \frac{C}{V} = \lambda \]
\[ \begin{bmatrix} C \\ V \end{bmatrix} = \begin{bmatrix} Q/V^2 \end{bmatrix} = \begin{bmatrix} \frac{AT}{M^2L^4T^{-1}} \\ \frac{A^2T^2} \end{bmatrix} = [M^{-2}L^{-4}T^{-7}A^3] \]

Chemistry

2. 2,4 DNP test can be used to identify

1. ether 2. aldehyde 3. halogens 4. amine

Solution:

2,4 DNP test is used to identify aldehydes and Ketones

Hence, Option 2 is correct.

7. Which pair of oxides is acidic in nature?


Solution:

Correct option 2 ) $B_2O_3$, $SiO_3$ are acidic and they form boric acid $H_3BO_3$ and silicic acid $H_4SiO_4$ on reaction with water.
10. The correct order of electron gain enthalpy is

1. O>S>Se>Te  
2. S>Se>Te>O  
3. Te>Se>S>O  
4. S>O>Se>Te

Solution:

The correct order of electron gain enthalpy is

\[ S > Se > Te > O \]

Due to the small size of Oxygen, it has the least negative value of electron gain enthalpy.

Hence, option 2 is correct.

13. In \( \text{molecule} \) the hybridization of carbon 1,2,3 and 4 respective are

1. sp\(^2\), sp\(^2\), sp\(^3\)  
2. sp\(^2\), sp\(^2\), sp\(^2\), sp\(^3\)  
3. sp\(^3\), sp, sp\(^3\), sp\(^3\)  
4. sp\(^2\), sp\(^3\), sp\(^2\), sp\(^3\)

Solution:

Hybridisation of carbons

1 - sp\(^2\)  
2 - sp  
3 - sp\(^2\)  
4 - sp\(^3\)

Correct option is 1) sp\(^2\), sp, sp\(^2\), sp\(^3\)

17. Which of the following forms of hydrogen emits low energy of particles?

\[ \begin{array}{ll}
1. \text{Tritium} & 3\ H \\
2. \text{Proton} & 1\ H \\
3. \text{Protium} & \frac{1}{2} H \\
4. \text{Deuterium} & \frac{1}{2} H
\end{array} \]

Solution:

The correct option is 1) Tritium \( \frac{3}{1} H \)

Tritium is the only radioactive that emits low energy \( \beta^- \) particles.

20. Seliwanoff test and Xantha protein test are used of the identification - and - respectively

1) aldoses, ketoses  
2) ketoses, aldoses  
3) Proteins, ketoses
4) Ketoses, Proteins

Solution:

Seliwanoff's test is used to identify ketoses.

Xanthoproteic test is used to identify proteins.

Hence option d) ketoses, proteins is correct.

21) The average S-F bond energy in kJ mol\(^{-1}\) of \(SF_6\) is rounded off to the nearest integer (given) the values of standard enthalpy of formation of \(SF_6(g)\), \(S(g)\) and \(F(g)\) are -1100, 275, 80 kJmol\(^{-1}\):

Solution:

\[
\frac{1}{8} S_8(s) + 3 F_2(g) \rightarrow SF_6(g)
\]

\[
\frac{1}{8} S_8(s) \rightarrow S(g) \quad \Delta H = 275 \text{ KJ} \text{mol}^{-1}
\]

\[
3 F_2(g) \rightarrow 6 F(g) \quad \Delta H = 80 \text{ KJ} \text{mol}^{-1}
\]

\[
S(g) + 6 F(g) \rightarrow SF_6(g) \Delta H = x \text{ KJ} \text{mol}^{-1}
\]

According to Bond energy concept,

\[
\Delta H_f^0(S) + 6 \Delta H_f^0(F) - 6 \Delta H_{BE}(SF) = \Delta H_f^0(SF_6)
\]

\[
275 + 6 \times 80 - 6x = -1100
\]

\[
6x = 275 + 480 + 1100
\]

\[
x = 1855
\]

\[
x = 309.17 \text{ KJ} \text{mol}^{-1}
\]

22) If the activation energy of a reaction is 80.9 kJ mol\(^{-1}\), the fraction of molecules at 700 k having enough energy to reach to form products is \(e^{-x}\). The value of \(x\) is __ [ use \(R = 8.31 \text{ J.K}^{-1} \text{mol}^{-1}\)]

Solution:

Given,

\[
E_a = 80.4 \text{ KJ} \text{mol}^{-1}
\]

\[
R = 8.31
\]

\[
T = 700 \text{ K}
\]

\[
\therefore \text{The fraction of molecules having energy greater than activation energy} = e^{\frac{-E_a}{RT}}
\]

\[
x = \frac{E_a}{RT} = \frac{80.4 \times 10^3}{8.31 \times 700} = 13.82
\]

23) The number of stereoisomers possible for \([Co(ox)_2(Br)(NH_3)]_i\) is __ [ox = oxalate]
Solution:

\([\text{Co(ox)}_2\text{Br(NH}_3)\text{]}\) is an octahedral compound it has 3 stereoisomers.

24) The pH of ammonium phosphate solution if \(\text{PK}_a\) of phosphoric acid and \(\text{PK}_b\) of ammonium hydroxide are 5.23 and 4.75 respectively is __

Solution:

pH of a salt of a weak acid and weak base is given by

\[
pH = 7 + \frac{1}{2}(pK_a - pK_b)
\]

According to given data,

\[
pH = 7 + \frac{1}{2}(5.23 - 4.75) = 7.24
\]

25. A ball weighing 10g is moving with a velocity of an uncertainty in its velocity is 5%. Then the uncertainty in its __ \(\times 10^{-33}\) (Given : \(\hbar = 6.63 \times 10^{-34}\text{ J} \cdot \text{s}\))

Solution:

Given,

\[
m = 10\text{ g} = 10^{-2}\text{ kg}
\]

\[
v = 90\text{ ms}^{-1}
\]

\[
\Delta v = \frac{5}{100} \times 90 = 4.5\text{ ms}^{-1}
\]

\[
\Delta P = 4.5 \times 10^{-2}
\]

Now, according to Heisenberg’s Uncertainty Principle

\[
(\Delta x) \times (\Delta P) \geq \frac{\hbar}{4\pi}
\]

\[
(\Delta x) \geq \frac{\hbar}{4\pi \times (\Delta P)}
\]
\[
\Delta x \geq \frac{6.63 \times 10^{-34}}{4 \times (3.14) \times (4.5 \times 10^{-34})} > 1.2 \times 10^{-34} \text{ m}
\]

Hence, the correct answer is 1.2

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**Maths**

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1. The triangle of maximum area that can be inscribed in a given circle of radius r is:

1. An equivalent triangle of height \( \frac{2r}{3} \)

2. An equilateral triangle having each of its side of length \( \sqrt{3r} \)

3. An isosceles triangle with base equal to 2r

4. A right angle traigle having two of its sides of length 2r and r.

2. Let abe an integer such that all the real roots of the polynomial \( 2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10 \) lie in the interval \((a, a+1)\). Then, \(|a|\) is equal to ___.

Solution:

Given polynomial equation has only one real root lying between (-2,-1)

Hence, \( a = -2 \)

\(|a| = 2\)

3. If \( m, n > 1 \) and \( \int_0^1 x^{m-1} + x^{n-1} dx = aI_{m,n} \), then a equals ___.

Solution

Given \( I(m, n) = \int_0^1 x^{m-1}(1 - x)^{n-1} \) dx

Now \( \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}} dx = aI(m, n) \)

\( \Rightarrow I = \int_0^1 \frac{x^{m-1}}{(1 + x)^{m+n}} dx + \int_0^1 \frac{x^{n-1}}{(1 + x)^{m+n}} dx \)
\[ \Rightarrow I = I_1 + I_2 \]

\[ I_2 = \int_0^1 \frac{x^{n-1}}{(1 + x)^{m+n}} \, dx \]

put \( x = \frac{1}{z} \)

\[ I_2 = \int_1^\infty \frac{z^{m-1}}{(1 + z)^{m+n}} \, dz = \int_1^\infty \frac{x^{m-1}}{(1 + x)^{m+n}} \, dx \]

From Above

\[ I = \int_0^1 \frac{x^{m-1}}{(1 + x)^{m+n}} \, dx + \int_1^\infty \frac{x^{m-1}}{(1 + x)^{m+n}} \, dx \]

\[ = \int_0^\infty \frac{x^{m-1}}{(1 + x)^{m+n}} \, dx = I(m, n) \]

a = 1

\[ f(x) = \int_1^x \frac{\log e}{1 + t} \, dt \]

6. For \( x > 0 \), if \( f(x) \), then \( F(e) + F\left(\frac{1}{e}\right) \) is equal to

1. 1
2. -1
3. \( \frac{1}{2} \)
4. 0

Solution

\[ f(x) = \int_1^x \frac{\ln t}{1 + t} \, dt \]

Now, \( f\left(\frac{1}{x}\right) = \int_{1/x}^1 \frac{\ln t}{1 + t} \, dt \)

Let \( t = \frac{1}{y} \Rightarrow dt = -\frac{1}{y^2} \, dy \)
7. The triangle of maximum area that can be inscribed in a given circle of radius $r$ is:

1. An equivalent triangle of height $2r$
2. An equilateral triangle having each of its sides of length $\sqrt{3}r$
3. An isosceles triangle with base equal to $2r$
4. A right angle triangle having two of its sides of length $2r$ and $r$.

Solution

Let the lowest point on the circle, with radius $R$, lie on the origin of the Cartesian coordinate axes.

Let us assume that the base $AB$ of $\triangle ABC$ is parallel to the $X$ axis. Let the coordinates of points $A$ and $B$ be $(-x,y)$ and $(x,y)$ respectively. In $\triangle ABC$, point $C$ must lie on the $Y$ axis and hence, the coordinates of point $C$ are $(0,2R)$.

The measures of the base and height of $\triangle ABC$ are $2x$ and $2R$ respectively.

The area of $\triangle ABC = x(2R-y)$.

The equation of the above circle is $x^2 + (y-R)^2 = R^2$

$x^2 = R^2 - (y^2 - 2yR + R^2) = 2yR - y^2$

$\Delta = \sqrt{2yR - y^2}(2R - y)$
Let $a$ and $b$ be positive two real numbers such that 
\[ \begin{align*} 
\frac{d\Delta}{dy} &= \frac{(R - y)(2R - y) - \sqrt{2yR - y^2}}{\sqrt{2yR - y^2}} = 0 \\
2R^2 - 3yR + y^2 - 2yR + y^2 &= 0 \\
2R^2 - 5yR + 2y^3 &= 0 \\
\sqrt{2yR - y^2} &= 0 \\
(2R - y)(R - 2y) &= 0 \\
\sqrt{y\sqrt{2R - y}} &= 0 \\
(R - 2y)\sqrt{2R - y} &= 0 \\
y &= 2R \quad \text{or} \quad y = \frac{R}{2} \\
y = \frac{R}{2}, \quad x = \sqrt{2yR - y^2} = \frac{\sqrt{3}}{2}R \\
AB = 2x = \sqrt{3}R \\
AC = BC = \left(\frac{\sqrt{3}}{2}R - 0\right)^2 + \left(2R - \frac{R}{2}\right)^2 = \sqrt{3}R
\end{align*} \]

1 Let $a$ and $b$ be positive two real numbers such that $\alpha + \beta = 1$ and $\alpha \beta = -1$, let 
\[ P_n = (\alpha)^n + (\beta)^n \]
$P_{n-1} = 11$ and $P_n + 1 = 29$ for some integer $n>1$. Then the value of $P_n^2$ is 

Solution

Given $\alpha + \beta = 1$ and $\alpha \beta = -1$, $P_{n-1} = 11$, $P_n = 29$

AM of $P_{n-1}$ and, $P_n = \frac{\alpha^{n-1} + \beta^{n-1} + \alpha \beta^n}{2}$

\[ = \frac{\alpha^{n-1}(\alpha + 1) + \beta^{n-1}(\beta + 1)}{2} \]

\[ = \frac{\alpha^{n-1}(\alpha^2) + \beta^{n-1}(\beta^2)}{2} \]

\(\therefore\) $\alpha^2 = \alpha + 1$ and $\beta^2 = \beta + 1$

\[ AM = \frac{1}{2} (\alpha^{n+1} + \beta^{n+1}) = \frac{1}{2}P_{n+1} \]

\[ \frac{P_{n-1} + P_n}{2} = \frac{1}{2}P_{n+1} \]

\[ \frac{11 + P_n}{2} = \frac{1}{2} \times 29 \Rightarrow P_n = 29 - 11 \]

\[ = 18 \]

Hence
\((P_n)^2 = (18^2)\)

\[= 324\text{ Ans}\]

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & 0 \\
3 & 0 & -1
\end{bmatrix}
\]

4. If the matrix satisfies the equation for some real numbers \(a\) and \(\beta\), then \(\beta - a\) is equal to \_

\[
A^{20} + aA^{19} + \beta A = \begin{bmatrix}
1 & 0 & 0 \\
0 & 4 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[ 1 + a + \beta = 1 \]
\[ a = -\beta \]
\[ 114 + 108a + 2\beta = 4 \]

5. Let L be a common tangent line to the curves \( 4x^2 - 9y^2 = 36 \) and \( (2x)^2 + (2y)^2 = 31 \). Then the square of the slope of the line L is___

Solution

\[ \frac{x^2}{9} + \frac{y^2}{4} = 1 \]
\[ x^2 + y^2 = \frac{31}{4} \]

Tangent of ellipse

\[ y = mx \pm a\sqrt{a^2m^2 + b^2} \]
\[ y = mx \pm a\sqrt{9m^2 + 4} \]

Tangent of circle

\[ y = mx \pm \sqrt{\frac{31}{4}} \sqrt{1 + m^2} \]

Since tangent is common

\[ \sqrt{9m^2 + 4} = \sqrt{\frac{31}{4}} \sqrt{1 + m^2} \]
\[ 36m^2 + 16 = 31 + 31m^2 \]
\[ m^2 = 3 \]

\[ f(x) = \int_0^t e^t dt + e^x \]

1) Let \( f(x) \) be a differentiable for all \( x \in \mathbb{R} \). Then \( f(x) \) equals.

1) \( e(e^x - 1) \)
2) \( (2e^x - 1) \)
3) \( 2e(e^x - 1) - 1 \)
4) \( e^x - 1 \)
Solution

\[ f(x) = \int_0^x e^t \, dt + e^x \]

\[ f(x) = [e^t]^x_0 + e^x \]

\[ f(x) = 2e^x - 1 \]

2) A natural number has Prime factorization given by \( n = 2^x 3^y 5^z \) where \( y \) and \( z \) are such that \( y + z = 5 \) and \( \frac{1}{y} + \frac{1}{z} = \frac{5}{6} ; y > z \). Then the number of odd divisors of \( n \) including 1, is:

1) 11
2) 6x
3) 12
4) 6

Solution

\[ n = 2^x 3^y 5^z \]

\[ y + z = 5 \]

\[ \frac{1}{y} + \frac{1}{z} = \frac{5}{6} \]

\[ y = 3 \]

\[ z = 2 \]

Answer: \( 1 \times 4 \times 3 = 12 \)

3) A seven digit number is formed using digits 3, 3, 4, 4, 5, 5. Then probability, that number so formed is divisible by 2, is:

\[ \frac{1}{7} \]

1) \( \frac{1}{7} \)
2) \( \frac{4}{7} \)
3) \( \frac{6}{7} \)
4) \( \frac{7}{7} \)

Solution
8) Let \( A (1, 4) \) and \( B (1, -5) \) be two points. Let \( P \) point on the circle \((x - 1)^2 + (y - 1)^2 = 1\) such that \((PA)^2 + (PB)^2\) have maximum value, then the points \( P, A \) and \( B \) lie on:

1) an ellipse
2) a parabola
3) a hyperbola
4) a straight line

Clearly it is straight line.

10) Let \( f(x) \) be a differentiable function at \( x = a \) with \( f(a) \) and \( f'(a) = 4 \) then \( \lim_{x \to a} \frac{x f(a) - a f(x)}{x - a} \)
1) $2a - 4$
2) $2a + 4$
3) $a + 4$
4) $4 - 2a$

Solution

$$\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$$

at $x = a$

L-Hospital

$$= \frac{f(a) - af'(a)}{1}$$

$$= 4 + 2a$$

20) If the focus of the mid-point of the line segment from the point $(3, 2)$ to a point on the circle, $x^2 + y^2 = 1$, is a circle of radius $r$ then $r$ is equal to

1) $\frac{1}{3}$
2) $\frac{1}{2}$
3) $\frac{1}{4}$
4) $1$

General point on the circle
$(\cos \theta, \sin \theta)$