15.5 MATHEMATICS (MA)

**Sequences and Series of Real Numbers:** Sequence of real numbers, convergence of sequences, bounded and monotone sequences, convergence criteria for sequences of real numbers, Cauchy sequences, subsequences, Bolzano-Weierstrass theorem. Series of real numbers, absolute convergence, tests of convergence for series of positive terms – comparison test, ratio test, root test; Leibniz test for convergence of alternating series.

**Functions of One Real Variable:** Limit, continuity, intermediate value property, differentiation, Rolle’s Theorem, mean value theorem, L’Hospital rule, Taylor’s theorem, maxima and minima.

**Functions of Two or Three Real Variables:** Limit, continuity, partial derivatives, differentiability, maxima and minima.

**Integral Calculus:** Integration as the inverse process of differentiation, definite integrals and their properties, fundamental theorem of calculus. Double and triple integrals, change of order of integration, calculating surface areas and volumes using double integrals, calculating volumes using triple integrals.

**Differential Equations:** Ordinary differential equations of the first order of the form \( y’ = f(x,y) \), Bernoulli’s equation, exact differential equations, integrating factor, orthogonal trajectories, homogeneous differential equations, variable separable equations, linear differential equations of second order with constant coefficients, method of variation of parameters, Cauchy-Euler equation.

**Vector Calculus:** Scalar and vector fields, gradient, divergence, curl, line integrals, surface integrals, Green, Stokes and Gauss theorems.

**Group Theory:** Groups, subgroups, Abelian groups, non-Abelian groups, cyclic groups, permutation groups, normal subgroups, Lagrange’s Theorem for finite groups, group homomorphisms and basic concepts of quotient groups.

**Linear Algebra:** Finite dimensional vector spaces, linear independence of vectors, basis, dimension, linear transformations, matrix representation, range space, null space, rank-nullity theorem. Rank and inverse of a matrix, determinant, solutions of systems of linear equations, consistency conditions, eigenvalues and eigenvectors for matrices, Cayley-Hamilton theorem.

**Real Analysis:** Interior points, limit points, open sets, closed sets, bounded sets, connected sets, compact sets, completeness of \( \mathbb{R} \). Power series (of real variable), Taylor’s series, radius and interval of convergence, term-wise differentiation and integration of power series.