PART: MATHEMATICS

1. How many four letters word can be formed with each 2 vowels and 2 consonants from word UNIVERSE?

Ans. 504

Sol. Vowels Consonants

L,R,E,E
N,V,R,S

Case-1

All are different \( \binom{5}{2} \times 3! \times 3! = 360 \) words

Case-2

2 are some + 2 are diff \( \binom{5}{2} \times \binom{4}{2} \times \frac{3!}{2!} = 180 \) words

Total No of words = 360 + 180 = 540

2. If \( f(x) = -|x| \) then \( \int_{-a}^{a} f(x) \sin(\pi x) \) is equal to

(1) \( x^2 \)  
(2) \( 2a^2 \)  
(3) \( 3a^2 \)  
(4) \( 4a^2 \)

Ans. (1)

Sol. Let \( I = \int_{-a}^{a} f(x) \sin(\pi x) dx \)  
\( I = \int_{-a}^{a} (-x) \sin(\pi x) dx \)  
\( I = \int_{-a}^{a} (x) \sin(\pi x) dx \)  
\( I = \int_{-a}^{a} x^2 \sin(\pi x) dx = 2a^2 \Rightarrow I = a^2 \)
2. The rank of the matrix
\[
\begin{bmatrix}
-1 & 0 & 2 & 3 \\
4 & 5 & -1 & 0 \\
2 & 3 & 0 & 1 \\
1 & 0 & 2 & 3
\end{bmatrix}
\]
is...

Ans. (4)

Sol.
\[
\begin{bmatrix}
-1 & 0 & 2 & 3 \\
4 & 5 & -1 & 0 \\
2 & 3 & 0 & 1 \\
1 & 0 & 2 & 3
\end{bmatrix}
\]
is invertible because its determinant is non-zero.

3. If \(x^2 + y^2 = 1\) and \(x^2 + y^2 = 9\), then the number of solutions for \((x, y)\) is...

Ans. (4)

Sol.
The two equations represent two circles. The number of solutions is determined by the relative positions of these circles.

4. If \(f(x) = \frac{1}{x^2} + 3x^2 - 4x + 1\) then the value of \(f(-2)\) is...

Ans. (2)

Sol.
\[
f(x) = \frac{1}{x^2} + 3x^2 - 4x + 1
\]
\[
f(-2) = \frac{1}{(-2)^2} + 3(-2)^2 - 4(-2) + 1 = \frac{1}{4} + 12 + 8 + 1 = 23
\]

5. If the eccentricity of an ellipse \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\) is reciprocal of the eccentricity of a hyperbola \(2x^2 - 4y^2 = 1\) and ellipse intersect hyperbola orthogonally, then the length of latus rectum of ellipse is...

Ans. \(\frac{2a^2}{\sqrt{2}}\)

Sol.
The eccentricity of the hyperbola is given by \(e^2 = \frac{a^2 - b^2}{a^2}\), and the ellipse by \(e = \sqrt{1 - \frac{b^2}{a^2}}\). Setting these equal gives the relationship to find the length of the latus rectum.

6. If there are 100 people in a group, in which 75 persons can read English and 40 persons can read Hindi. If 10 persons can read English only and 5 persons can read Hindi only, then the number of persons who can read both is...

Ans. (2)

Sol.
The total number of persons who can read at least one language is 100. The number who can read both languages is the difference of those who can read English and those who can read Hindi, after subtracting those who can read only one language.

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This solution was downloaded from Resonance JEE (Main) 2023 Solution portal PAGE # 1
Statement – 1: \((23)^{2022} \mod (1999)^{2022}\) is divisible by 9
Statement – 2: \((13)^{11} \mod 11\) is divisible by 144
(1) Statement 1 and statement 2 both correct
(2) Statement 1 is correct and statement 2 is incorrect
(3) Statement 1 is incorrect and statement 2 is correct
(4) Statement 1 and statement 2 both are incorrect.

Ans. (3)

Sol. Statement 1: \(x^{2022} \mod y^{2022}\) is divisible by \((x - y)\)
Hence \((23)^{2022} \mod (1999)^{2022}\) is divisible by 23 - 1999 = 24
\(\Rightarrow\) statement 1 is incorrect.
Statement 2: \(13^{11} \mod 11\) is divisible by 144
\(\Rightarrow\) Statement 2 is correct.

8. Sum of all values of \(\alpha\) for which points with position vectors \(-3\hat{i} + 2\hat{j} + 3\hat{k}\), \(2\hat{i} - 3\hat{j} + 4\hat{k}\), \((\alpha + 1)\hat{i} + \hat{j} + 2\hat{k}\) and \(9\hat{i} = (-\hat{j}) + 6\hat{k}\) are coplanar.

\(\begin{align*}
\alpha & = 1 \\
\alpha & = 1 \\
\alpha & = 6 \Rightarrow \exists
\end{align*}\)

Ans. (1)

Sol. Let \(A(3\hat{i} + 2\hat{j} + 3\hat{k})\), \(B(2\hat{i} - 3\hat{j} + 4\hat{k})\), \(C(\alpha + 1)\hat{i} + \hat{j} + 2\hat{k}\) and \(D(9\hat{i} = (-\hat{j}) + 6\hat{k})\)

\(\Rightarrow\) \(A, B, C, D\) are coplanar iff

\(\left[ \begin{array}{ccc}
1 & -1 & 1 \\
1 & 0 & 1 \\
6 & 0 & 1
\end{array} \right] = 0\)

\(\Rightarrow\) \(6a - 6 = 1(3a + 15) + 1\) \(a^2 - 6a - 16 = 0\)

\(\Rightarrow\) \(a = 4, 4\)

9. If \(P = l(lP \pm lP)\), then \(P = l\pm 13P\) then value of \(\alpha\) is \(\alpha = \alpha\).

Ans. 4

Sol. \(p = P = l = 13P\)

\(\Rightarrow\) \(P = l\pm 13P\)

\(\Rightarrow\) \(2P = l\pm 42P\)

\(\Rightarrow\) \(P = l\pm 2P\)...

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12. If volume of a parallelepiped whose coterminous edges are \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) is \( V \) then volume of parallelepiped whose coterminous edges are \( \mathbf{a} + \mathbf{b} + \mathbf{c}, \mathbf{a} + 2\mathbf{b} + 3\mathbf{c} \) is

\[
\begin{align*}
(1) & \quad \frac{1}{2} \\
(2) & \quad \frac{V}{3} \\
(3) & \quad \frac{V}{9} \\
(4) & \quad V
\end{align*}
\]

Ans. (4)

Scl. Since \( V = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \)

Now new volume is \( V' = (\mathbf{a} + \mathbf{b} + \mathbf{c}) \cdot ((\mathbf{a} + 2\mathbf{b} + 3\mathbf{c}) \times (\mathbf{a} + \mathbf{b} + \mathbf{c})) \)

13. Let three dice are thrown. If the probability that all three dice show distinct numbers is \( \frac{p}{q} \) then value of \( p - q \) is

\( q - p \) is \( \ldots \) (where HCF \( (p, q) = 1 \) )

Ans. (4)

Scl. Total number of outcomes \( = 6 \times 6 \times 6 \)

Total number of favourable outcomes \( = 6 \times 5 \times 4 \)

So probability \( = \frac{5}{9} \)

14. If a plane passes through the line of intersection of planes \( x + y + z = 6 \) and \( x + (2z - 4) = 5 \).

If this plane passes through \( P(0, 2, -2) \), then square of distance of point \((12, 12, 18)\) from plane is \( \ldots \)

Scl.

Given planes

\[
\begin{align*}
P_1 : & \quad x + y + z = 6 \\
P_2 : & \quad 2x + 3y + 4z = 5
\end{align*}
\]

Equation of plane passing through line of intersection of \( P_1 = 0 \) & \( P_2 = 0 \) is \( P_1 + \lambda P_2 = 0 \)

\[
\begin{align*}
(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z = 0 \\
\text{It passes through (0, 2, -2)} \Rightarrow 2(1 + 4\lambda) = 0 \\
2 = -8\lambda \Rightarrow \lambda = \frac{1}{4}
\end{align*}
\]

So plane is \( 5x + 7y + 9z + 4 = 0 \)

Distance of \((12, 12, 18)\) from plane is

\[
\begin{align*}
d & = \frac{5 \times (12) + 7 \times (12) + 9 \times (18) + 4}{\sqrt{5^2 + 7^2 + 9^2}} \\
& = \frac{310}{1155} \\
d & = 4 \times \frac{155}{1155} = \frac{155}{280}
\end{align*}
\]
16. Statement -1 
(p \Rightarrow q) \lor (\neg p \land q) is a tautology.
Statement -2 
(q \Rightarrow p) \lor (\neg q \land p) is a contradiction.
(1) only statement -1 is correct
(2) only statement -2 is correct
(3) both statements are correct
(4) both statements are incorrect
Ans. (4)
Sol.
Statement -1: 
(p \Rightarrow q) \lor (\neg p \land q)
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Statement -2: 
(q \Rightarrow p) \lor (\neg q \land p)
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Statement -1 is not tautology.
Statement -2 is a contradiction.