



**JEE
MAIN
FEB.
2021**

24th Feb. 2021 | Shift - 2
MATHEMATICS

JEE | NEET | Foundation

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SELECTIONS SINCE 2007

1. Let $a, b \in \mathbb{R}$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line

$$\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9} \text{ is } (20, b, -a-9), \text{ then } |a+b| \text{ is equal to :}$$

(1) 86

(2) 88

(3) 84

(4) 90

Ans. (2)

Sol. $P(a, 6, 9), Q(20, b, -a-9)$

$$\text{mid point of } PQ = \left(\frac{a+20}{2}, \frac{b+6}{2}, \frac{-a}{2} \right)$$

lie on line

$$\frac{\frac{a+20}{2}-3}{7} = \frac{\frac{b+6}{2}-2}{5} = \frac{\frac{-a}{2}-1}{-9}$$

$$\frac{a+20-6}{14} = \frac{b+6-4}{10} = \frac{-a-2}{-18}$$

$$\frac{a+14}{14} = \frac{a+2}{18}$$

$$18a + 252 = 14a + 28$$

$$4a = -224$$

$$\boxed{a = -56}$$

$$\frac{b+2}{10} = \frac{a+2}{18}$$

$$\frac{b+2}{10} = \frac{-54}{18}$$

$$\frac{b+2}{10} = -3 \Rightarrow b = -32$$

$$|a+b| = |-56-32| = 88$$

2. Let f be a twice differentiable function defined on \mathbb{R} such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$

for all $x \in \mathbb{R}$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in \mathbb{R}$ then the value of $f(1)$ lies in the interval:

(1) (9, 12)

(2) (6, 9)

(3) (3, 6)

(4) (0, 3)

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Ans. (2)

Sol. Given $f(x) f''(x) - (f'(x))^2 = 0$

$$\text{Let } h(x) = \frac{f(x)}{f'(x)}$$

$$\Rightarrow h'(x) = 0 \quad \Rightarrow h(x) = k$$

$$\Rightarrow \frac{f(x)}{f'(x)} = k \quad \Rightarrow f(x) = k f'(x)$$

$$\Rightarrow f(0) = k f'(0) \quad \Rightarrow 1 = k(2) \Rightarrow k = \frac{1}{2}$$

$$\text{Now } f(x) = \frac{1}{2} f'(x) \Rightarrow \int 2 dx = \int \frac{f'(x)}{f(x)} dx$$

$$\Rightarrow 2x = \ln|f(x)| + C$$

$$\text{As } f(0) = 1 \Rightarrow C = 0$$

$$\Rightarrow 2x = \ln|f(x)| \Rightarrow f(x) = \pm e^{2x}$$

$$\text{As } f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$$

3. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:

(1) $\frac{1}{2\sqrt{2}}$

(2) $\frac{1}{\sqrt{7}}$

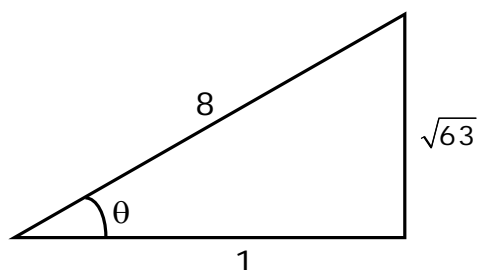
(3) $\sqrt{7} - 1$

(4) $2\sqrt{2} - 1$

Ans. (2)

Sol. $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

$$\text{Let } \sin^{-1}\left(\frac{\sqrt{63}}{8}\right) = \theta \quad \sin \theta = \frac{\sqrt{63}}{8}$$



$$\cos \theta = \frac{1}{8}$$

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$$2\cos^2\frac{\theta}{2}-1=\frac{1}{8}$$

$$\cos^2\frac{\theta}{2}=\frac{9}{16}$$

$$\cos\frac{\theta}{2}=\frac{3}{4}$$

$$\frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}}=\frac{3}{4}$$

$$\tan\frac{\theta}{4}=\frac{1}{\sqrt{7}}$$

4. The probability that two randomly selected subsets of the set $\{1,2,3,4,5\}$ have exactly two elements in their intersection, is:

(1) $\frac{65}{2^7}$

(2) $\frac{135}{2^9}$

(3) $\frac{65}{2^8}$

(4) $\frac{35}{2^7}$

Ans. (2)

Sol. Required probability

$$= \frac{{}^5C_2 \times 3^3}{4^5}$$

$$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$$

5. The vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$, and the point $(1,0,2)$ is :

(1) $\vec{r} \cdot (\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

(2) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(3) $\vec{r} \cdot (3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(4) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

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Ans. (2)

Sol. Plane passing through intersection of plane is

$$\{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1\} + \lambda \{\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2\} = 0$$

Passes through $\hat{i} + 2\hat{k}$, we get

$$(3 - 1) + \lambda(1 + 2) = 0 \Rightarrow \lambda = -\frac{2}{3}$$

Hence, equation of plane is $3\{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1\} - 2\{\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2\} = 0$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

6. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(1) (-2, 8)

(2) (1, 5)

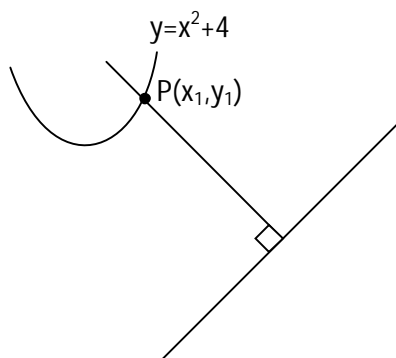
(3) (3, 13)

(4) (2, 8)

Ans. (4)

Sol. $\frac{dy}{dx} \Big|_P = 4$

$$\therefore 2x_1 = 4$$



$$\Rightarrow x_1 = 2$$

\therefore Point will be (2, 8)

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7. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a, c), (2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is:
- (1) $\frac{71}{256}$ (2) $-\frac{69}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{71}{256}$

Ans. (4)

Sol. $2b = a + c$
 $\frac{2a+2}{3} = \frac{10}{3}$ and $\frac{2b+c}{3} = \frac{7}{3}$
 $a = 4, \begin{cases} 2b+c=7 \\ 2b-c=4 \end{cases}$, solving
 $b = \frac{11}{4}$
 $c = \frac{3}{2}$

\therefore Quadratic Equation is $4x^2 + \frac{11}{4}x + 1 = 0$

\therefore The value of $(\alpha + \beta)^2 - 3\alpha\beta = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$

8. The value of the integral, $\int_1^3 [x^2 - 2x - 2] dx$, where $[x]$ denotes the greatest integer less than or equal to x , is:
- (1) -4 (2) -5 (3) $-\sqrt{2} - \sqrt{3} - 1$ (4) $-\sqrt{2} - \sqrt{3} + 1$

Ans. (3)

Sol. $I = \int_1^3 -3dx + \int_1^3 [(x-1)^2] dx$
 Put $x-1 = t$; $dx = dt$
 $I = (-6) + \int_0^2 [t^2] dt$
 $I = -6 + \int_0^1 0 dt + \int_1^{\sqrt{2}} 1 dt + \int_{\sqrt{2}}^{\sqrt{3}} 2 dt + \int_{\sqrt{3}}^2 3 dt$
 $I = -6 + (\sqrt{2} - 1) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$
 $I = -1 - \sqrt{2} - \sqrt{3}$

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11. The negation of the statement

$\sim p \wedge (p \vee q)$ is :

- (1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim p \vee q$ (4) $p \vee \sim q$

Ans. (4)

Sol.

p	q	$\sim p$	$p \vee q$	$(\sim p) \wedge (p \vee q)$	$\sim q$	$p \vee \sim q$
T	T	F	T	F	F	T
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	T	T

$\therefore \sim p \wedge (p \vee q) \equiv p \vee \sim q$

12. For the system of linear equations:

$$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbf{R}$$

consider the following statements:

- (A) The system has unique solution if $k \neq 2, k \neq -2$.
 (B) The system has unique solution if $k = -2$.
 (C) The system has unique solution if $k = 2$.
 (D) The system has no-solution if $k = 2$.
 (E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct?

- (1) (B) and (E) only (2) (C) and (D) only
 (3) (A) and (D) only (4) (A) and (E) only

Ans. (3)

Sol. $x - 2y + 0.z = 1$

$$x - y + kz = -2$$

$$0.x + ky + 4z = 6$$

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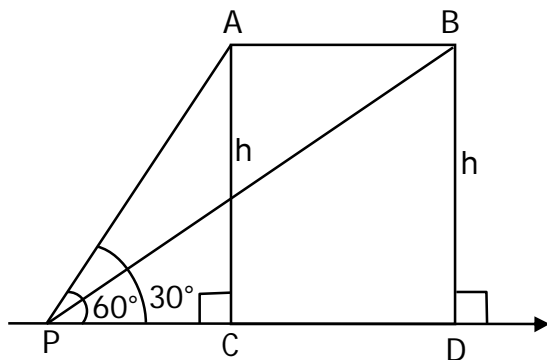
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14. The angle of elevation of a jet plane from a point A on the ground is 60° . After a flight of 20 seconds at the speed of 432 km/ hour, the angle of elevation changes to 30° . If the jet plane is flying at a constant height, then its height is:

- (1) $1200\sqrt{3}\text{m}$ (2) $1800\sqrt{3}\text{m}$ (3) $3600\sqrt{3}\text{m}$ (4) $2400\sqrt{3}\text{m}$

Ans. (1)

Sol.



$$v = 432 \times \frac{1000}{60 \times 60} \text{ m/sec} = 120 \text{ m/sec}$$

$$\text{Distance } AB = v \times 20 = 2400 \text{ meter}$$

In ΔPAC

$$\tan 60^\circ = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$$

In ΔPBD

$$\tan 30^\circ = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$$

$$PD = PC + CD$$

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$$

$$h = 1200 \sqrt{3} \text{ meter}$$

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15. For the statements p and q , consider the following compound statements:

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow p$

Then which of the following statements is correct?

- (1) (a) is a tautology but not (b) (2) (a) and (b) both are not tautologies.
(3) (a) and (b) both are tautologies. (4) (b) is a tautology but not (a).

Ans. (3)

	p	q	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim p$	$(\sim q) \wedge (p \rightarrow q) \rightarrow \sim p$
Sol. (a)	T	T	F	T	F	F	T
	T	F	T	F	F	F	T
	F	T	F	T	F	T	T
	F	F	T	T	T	T	T

(a) is tautologies

	p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$((p \vee q) \wedge \sim p) \rightarrow q$
(b)	T	T	T	F	F	T
	T	F	T	F	F	T
	F	T	T	T	T	T
	F	F	F	T	F	T

(b) is tautologies

\therefore a & b are both tautologies.

16. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2 B^2 - B^2 A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

- (1) a unique solution (2) exactly two solutions
(3) infinitely many solutions (4) no solution

Ans. (3)

Sol. $A^T = A, B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$$

$$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$$

$$= B^2A^2 - A^2B^2$$

$\Rightarrow P$ is skew-symmetric matrix

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore ay + bz = 0 \quad \dots(1)$$

$$-ax + cz = 0 \quad \dots(2)$$

$$-bx - cy = 0 \quad \dots(3)$$

From equation 1,2,3

$$\Delta = 0 \text{ \& } \Delta_1 = \Delta_2 = \Delta_3 = 0$$

\therefore equation have infinite number of solution

17. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$ is :

$$(1) \frac{n(n+1)^2(n+2)}{12}$$

$$(2) \frac{n(n-1)(2n+1)}{6}$$

$$(3) \frac{n(n+1)(2n+1)}{6}$$

$$(4) \frac{n(2n+1)(3n+1)}{6}$$

Ans. (3)

Sol. ${}^2C_2 = {}^3C_3$

$$S = {}^3C_3 + {}^3C_2 + \dots + {}^nC_2 = {}^{n+1}C_3$$

$$\therefore {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$$

$$= \frac{(n+1)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$$

$$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)(2n+1)}{6}$$

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18. If a curve $y = f(x)$ passes through the point $(1,2)$ and satisfies $x \frac{dy}{dx} + y = bx^4$, then for what

value of $b, \int_1^2 f(x)dx = \frac{62}{5}$?

- (1) 5 (2) $\frac{62}{5}$ (3) $\frac{31}{5}$ (4) 10

Ans. (4)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$, I.F. = $e^{\int \frac{dx}{x}} = x$

$$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + C$$

Passes through $(1,2)$, we get

$$2 = \frac{b}{5} + C \dots (i)$$

$$\text{Also, } \int_1^2 \left(\frac{bx^4}{5} + \frac{c}{x} \right) dx = \frac{62}{5}$$

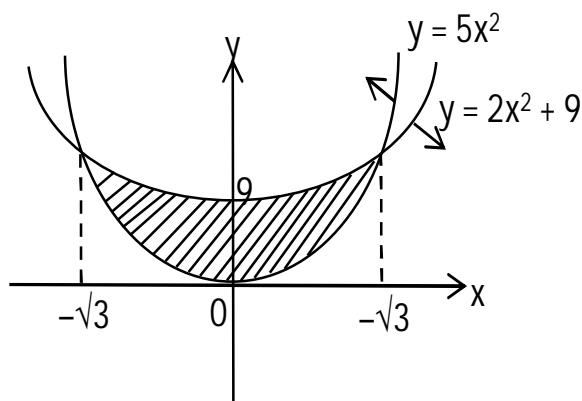
$$\Rightarrow \frac{b}{25} \times 32 + C \ln 2 - \frac{b}{25} = \frac{62}{5} \Rightarrow C = 0 \text{ \& } b = 10$$

19. The area of the region : $R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$ is:

- (1) $9\sqrt{3}$ square units (2) $12\sqrt{3}$ square units (3) $11\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

Ans. (2)

Sol.



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Required area

$$\begin{aligned}
 &= 2 \int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx \\
 &= 2 \int_0^{\sqrt{3}} (9 - 3x^2) dx \\
 &= 2 [9x - x^3]_0^{\sqrt{3}} = 12\sqrt{3}
 \end{aligned}$$

20. Let $f(x)$ be a differentiable function defined on $[0,2]$ such that $f'(x) = f'(2-x)$ for all $x \in (0,2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x) dx$ is:

- (1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1 - e^2)$ (4) $2(1 + e^2)$

Ans. (1)

Sol. $f'(x) = f'(2-x)$

On integrating both side $f(x) = -f(2-x) + c$

put $x = 0$

$$f(0) + f(2) = c \quad \Rightarrow c = 1 + e^2$$

$$\Rightarrow f(x) + f(2-x) = 1 + e^2 \dots\dots(i)$$

$$I = \int_0^2 f(x) dx = \int_0^1 \{f(x) + f(2-x)\} dx = (1 + e^2)$$

Section B

1. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is _____.

Ans. 2

Sol. $x \geq 5$

$$(x+1)^2 + (x-5) = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - \frac{43}{4} = 0$$

$$\Rightarrow 4x^2 + 12x - 43 = 0$$

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$$x = \frac{-12 \pm \sqrt{144 + 688}}{8}$$

$$x = \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$$

$$= \frac{-3 \pm 7.2}{2}$$

$$= \frac{-3+7.2}{2}, \frac{-3-7.2}{2} \text{ (Therefore no solution)}$$

For $x \leq 5$

$$(x+1)^2 - (x-5) = \frac{27}{4}$$

$$x^2 + x + 6 - \frac{27}{4} = 0$$

$$4x^2 + 4x - 3 = 0$$

$$x = \frac{-4 \pm \sqrt{16+48}}{8}$$

$$x = \frac{-4 \pm 8}{8} \Rightarrow x = -\frac{12}{8}, \frac{4}{8}$$

\therefore 2 Real Root's

2. The students S_1, S_2, \dots, S_{10} are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is_____.

Ans. 31650

Sol.

$$C \rightarrow 1 \quad 9 \begin{cases} A \\ B \end{cases}$$

$$C \rightarrow 2 \quad 8 \begin{cases} A \\ B \end{cases}$$

$$C \rightarrow 3 \quad 7 \begin{cases} A \\ B \end{cases}$$

$$= {}^{10}C_1 [2^9 - 2] + {}^{10}C_2 [2^8 - 2] + {}^{10}C_3 [2^7 - 2]$$

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$$\begin{aligned}
 &= 2^7 [{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240 \\
 &= 128 [40 + 90 + 120] - 350 \\
 &= (128 \times 250) - 350 \\
 &= 10[3165] = 31650
 \end{aligned}$$

3. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \neq 0$, then the value of the expression

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} \text{ is } \underline{\hspace{2cm}}.$$

Ans. 2

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ (i)

$$x \rightarrow \frac{1}{x}$$

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$$
(ii)

(i) + (ii)

$$(a + \alpha) \left[f(x) + f\left(\frac{1}{x}\right) \right] = \left(x + \frac{1}{x} \right) (b + \beta)$$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

4. If the variance of 10 natural numbers $1, 1, 1, \dots, 1, k$ is less than 10, then the maximum possible value of k is _____.

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Ans. 11

Sol. $\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$$\sigma^2 = \frac{(9+k^2)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$$

$$(90 + k^2) 10 - (81 + k^2 + 8k) < 1000$$

$$90 + 10k^2 - k^2 - 18k - 81 < 1000$$

$$9k^2 - 18k + 9 < 1000$$

$$(k-1)^2 < \frac{1000}{9} \Rightarrow k-1 < \frac{10\sqrt{10}}{3}$$

$$k < \frac{10\sqrt{10}}{3} + 1$$

Maximum integral value of k = 11

5. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and

$x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

Ans. 1

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$$\frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{2}{-1} \quad \dots(1)$$

Point on line = $\left(\lambda, \frac{1}{2}, 0\right)$

$$\frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1} \quad \dots(2)$$

Point on line = $(0, -2\lambda, \lambda)$

$$\text{Distance between skew lines} = \frac{[\vec{a}_2 - \vec{a}_1 \quad \vec{b}_1 \quad \vec{b}_2]}{|\vec{b}_1 \times \vec{b}_2|}$$

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$$\begin{vmatrix} \lambda & \frac{1}{2} + 2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{\left| -5\lambda - \frac{3}{2} \right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}} \text{ (given)}$$

$$= |10\lambda + 3| = 7 \Rightarrow \lambda = -1$$

$$\Rightarrow |\lambda| = 1$$

6. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and $n = \llbracket |k| \rrbracket$ be the greatest integral part of $|k|$.

Then $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$ is equal to _____.

Ans. 310

Sol.
$$\frac{\left(2e^{\frac{i2\pi}{3}} \right)^{21}}{\left(\sqrt{2}e^{-\frac{i\pi}{4}} \right)^{24}} + \frac{\left(2e^{\frac{i\pi}{3}} \right)^{21}}{\left(\sqrt{2}e^{\frac{i\pi}{4}} \right)^{24}}$$

$$\Rightarrow \frac{2^{21} \cdot e^{i14\pi}}{2^{12} \cdot e^{-i6\pi}} + \frac{2^{21} (e^{i7\pi})}{2^{12} (e^{i6\pi})}$$

$$\Rightarrow 2^9 e^{i(20\pi)} + 2^9 e^{i\pi}$$

$$\Rightarrow 2^9 + 2^9(-1) = 0$$

$$n = 0$$

$$\sum_{j=0}^5 (j+5)^2 - \sum_{j=0}^5 (j+5)$$

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$$\begin{aligned} &\Rightarrow [5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2] - [5 + 6 + 7 + 8 + 9 + 10] \\ &\Rightarrow [(1^2 + 2^2 + \dots + 10^2) - (1^2 + 2^2 + 3^2 + 4^2)] - [(1 + 2 + 3 + \dots + 10) - (1 + 2 + 3 + 4)] \\ &\Rightarrow (385 - 30) - [55 - 10] \\ &\Rightarrow 355 - 45 \Rightarrow 310 \text{ ans.} \end{aligned}$$

7. Let a point P be such that its distance from the point (5,0) is thrice the distance of P from the point (-5,0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to

Ans. 56.25

Sol. Let P(h,k)

Given

$$PA = 3PB$$

$$PA^2 = 9PB^2$$

$$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2 + k^2]$$

$$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$$

\therefore Locus

$$x^2 + y^2 + \left(\frac{25}{2}\right)x + 25 = 0$$

$$\therefore c \equiv \left(\frac{-25}{4}, 0\right)$$

$$\therefore r^2 = \left(\frac{-25}{4}\right)^2 - 25$$

$$= \frac{625}{16} - 25$$

$$= \frac{225}{16}$$

$$\therefore 4r^2 = 4 \times \frac{225}{16} = \frac{225}{4} = 56.25$$

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8. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^n C_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$$\sum_{i=0}^k \binom{10}{i} \binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+1-i} \text{ exists, is equal to } \underline{\hspace{2cm}}.$$

Ans. 12

Sol. $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \dots + {}^{10}C_{10}x^{10}$

$$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \dots + {}^{15}C_{k-1}x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1} + \dots + {}^{15}C_{15}x^{15}$$

$$\sum_{i=0}^k ({}^{10}C_i)({}^{15}C_{k-i}) = {}^{10}C_0 \cdot {}^{15}C_k + {}^{10}C_1 \cdot {}^{15}C_{k-1} + \dots + {}^{10}C_k \cdot {}^{15}C_0$$

Coefficient of x_k in $(1+x)^{25}$
 $= {}^{25}C_k$

$$\sum_{i=0}^{k+1} ({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0 \cdot {}^{13}C_{k+1} + {}^{12}C_1 \cdot {}^{13}C_k + \dots + {}^{12}C_{k+1} \cdot {}^{13}C_0$$

Coefficient of x^{k+1} in $(1+x)^{25}$
 $= {}^{25}C_{k+1}$
 ${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

For maximum value

$$k+1 = 13$$

$$k = 12$$

9. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α , then 2α is $\underline{\hspace{2cm}}$.

Ans. 3

Sol. a, ar, ar^2, ar^3

$$a + ar + ar^2 + ar^3 = \frac{65}{12} \quad \dots\dots\dots(1)$$

$$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$$

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$$\frac{1}{a} \left(\frac{r^3 + r^2 + r + 1}{r^3} \right) = \frac{65}{18} \quad \dots\dots\dots(2)$$

$$\frac{(i)}{(ii)}, a^2 r^3 = \frac{18}{12} = \frac{3}{2}$$

$$a^3 r^3 = 1 \Rightarrow a \left(\frac{3}{2} \right) = 1 \Rightarrow a = \frac{2}{3}$$

$$\frac{4}{9} r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$$

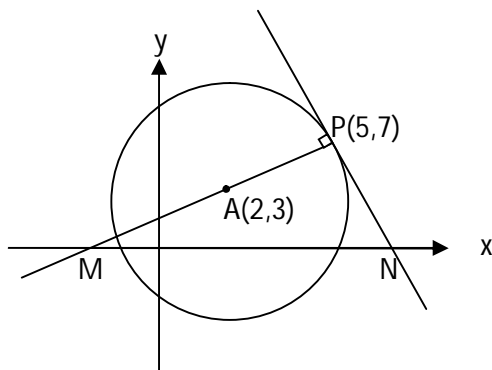
$$\alpha = ar^2 = \frac{2}{3} \cdot \left(\frac{3}{2} \right)^2 = \frac{3}{2}$$

$$2\alpha = 3$$

10. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point $(5,7)$ is A, then $24A$ is equal to_____.

Ans. 1225

Sol.



Equation of normal at P

$$(y - 7) = \left(\frac{7 - 3}{5 - 2} \right) (x - 5)$$

$$3y - 21 = 4x - 20$$

$$\Rightarrow 4x - 3y + 1 = 0 \quad \dots\dots\dots(i)$$

$$\Rightarrow M \left(-\frac{1}{4}, 0 \right)$$

Equation of tangent at P

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$$(y - 7) = -\frac{3}{4}(x - 5)$$

$$4y - 28 = -3x + 15$$

$$\Rightarrow 3x + 4y = 43 \quad \dots\dots\dots(ii)$$

$$\Rightarrow N\left(\frac{43}{3}, 0\right)$$

$$\text{Hence ar } (\Delta PMN) = \frac{1}{2} \times MN \times 7$$

$$\lambda = \frac{1}{2} \times \frac{175}{12} \times 7$$

$$\Rightarrow 24\lambda = 1225$$

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