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PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

(Reproduced from memory retention)

Date : 25 February, 2021 (SHIFT-2) Time ; (3.00 pm to 6.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

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MATHEMATICS

1. If $z^2 + \alpha z + \beta = 0$ has one root $1 - 2i$, where $\alpha, \beta, \in R$ find the value of $\alpha - \beta$

(1) 12 (2) 10 (3) –7 (4) 7

Ans. (3)

Sol. If the root is $1 - 2i$, the other roots is $1 + 2i$

Sum = 2, Product = 5

$\therefore$ quadratic equation $z^2 - 2z + 5 = 0$

$\Rightarrow \alpha = -2, \beta = 5$

$\alpha - \beta = -2 - 5 = -7$

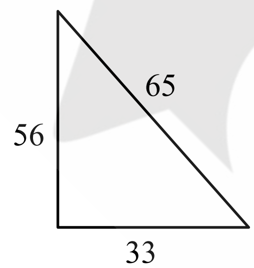
2. Find the value of $\operatorname{cosec}\left(2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

(1) $\frac{65}{56}$ (2) $\frac{56}{65}$ (3) $\frac{63}{65}$ (4) $\frac{65}{63}$

Ans. (1)

Sol. $= \operatorname{cosec}\left(\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right)$

$= \operatorname{cosec}\left(\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right)$



$= \operatorname{cosec}\left(\tan^{-1}\left(\frac{56}{33}\right)\right)$

$= \frac{65}{56}$

3. If $a_n = \alpha^n - \beta^n$ and α, β are the roots of the equation $x^2 - 6x - 2 = 0$ find the value of $\frac{a_{10} - 2a_8}{3a_9}$

(1) 2 (2) –2 (3) 3 (4) –3

Ans. (1)

Sol. $E = \frac{(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)}{3(\alpha^9-\beta^9)}$

$= \frac{\alpha^8(\alpha^2-2)-\beta^8(\beta^2-2)}{3(\alpha^9-\beta^9)}$

$= \frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{3(\alpha^9-\beta^9)} = \frac{6(\alpha^9-\beta^9)}{3(\alpha^9-\beta^9)} = 2$

4. If $f(x) = \frac{5^x}{5+5^x}$ find $f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+........+f\left(\frac{39}{20}\right)$

(1) 20 (2) $\frac{29}{2}$ (3) $\frac{19}{2}$ (4) $\left(\frac{39}{2}\right)$

Ans. (4)

Sol. $f(x) = \frac{5^x}{5+5^x}$

$f(2-x) = \frac{5^{2-x}}{5+5^{2-x}}$

$= \frac{25}{5.5^x+25} = \frac{5}{5^x+5}$

$f(x) + f(2-x) = 1$

Now $\left(f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)\right) + \left(f\left(\frac{2}{20}\right)+f\left(\frac{38}{20}\right)\right) + ...\left(f\left(\frac{19}{12}\right)+f\left(\frac{21}{20}\right)\right) + f\left(\frac{20}{20}\right)$

$= 1 \times 19 + \frac{1}{2} = \frac{39}{2}$

5. $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{n}{(n+1)}+\frac{n}{(n+2)^2}+........\frac{n}{(2n-1)^2}\right)$ is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) 1 (4) $\frac{2}{3}$

Ans. (1)

Sol. $1 = \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}$

$\therefore L = \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\left(\frac{r}{n}\right)^2+2\left(\frac{r}{n}\right)+1}$

$\therefore L = \int_0^1\frac{dx}{(x+1)^2} = \left.\frac{-1}{x+1}\right|_0^1$

$L = -\frac{1}{2} + 1 = \frac{1}{2}$

6. The system 2x + 3y + 2z = 1, 4x + 6y + 2z = 1, – x + y + 2z = 3 has

(1) unique solution (2) no solution (3) infinite solution (4) none of these

Ans. (1)

Sol. $D = \begin{vmatrix} 2 & +3 & 2 \\ 4 & 6 & 2 \\ -1 & 1 & 2 \end{vmatrix} = 2(10) - 3(10) + 2(10) \neq 0$

so unique solution

7. If A is a 3 × 3 matrix and |A| = 4 and R_i denote the i^{th} row of A, if matrix B is formed using $R_2 \to 2R_2 + 5R_3$ in matrix 2A then find |B|.

(1) 64 (2) 80 (3) 128 (4) 16

Ans. (1)

Sol. |A| = 4

$|2A| = 2^3\,|A| = 8 \times 4$

Now $R_2 \to 2R_2 + 5R_3$

|B| = 2 × 32

= 64

8. $\lim\limits_{x\to 0}\dfrac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)} = b$ if limit exist then find 2(a + b) ?

(1) –1 (2) –7 (3) 1 (4) 7

Ans. (4)

Sol. $\lim\limits_{x\to 0}\dfrac{ax-\left(e^{4x}-1\right)}{ax.4x}$

Apply L' Hospital

$\lim\limits_{x\to 0}\dfrac{a-\left(e^{4x}\right).4}{8ax}$ ($\dfrac{a-4}{0}$ form)

4 limit exist a = 4

$\lim\limits_{x\to 0}\dfrac{4-4e^{4x}}{32x} = \lim\limits_{x\to 0}\dfrac{1-e^{4x}}{8x} = \dfrac{-1}{2}$

a = 4 , b = $\dfrac{-1}{2}$

2 (a + b) = 2 $\left(4-\dfrac{1}{2}\right) = 7$

9. A is the set of all four digit natural numbers which has exactly one digit '7'. Find the probability of choosing a number from the set A so that it leaves the remainder 2 when divided by 5.

(1) $\frac{97}{297}$ (2) $\frac{91}{297}$ (3) $\frac{37}{297}$ (4) None of these

Ans. (1)

Sol. $n(A) = 7......... + \underbrace{..................}_{7}$

$= 1 \times 9 \times 9 \times 9 + 8 \times {}^3C_1 \times 1 \times 9 \times 9$

$= 729 + 1944 = 2673$

Favourable : $...........7 + \underbrace{................2}_{\text{7 exavtly once}}$

$= 8 \times 9 \times 9 + 1 \times 9 \times 9 \times 1 + 2 \times 8 \times 1 \times 9$

$= 648 + 81 + 144$

$= 873$

$\therefore$ Probability $= \frac{873}{2673} = \frac{97}{297}$

10. Minimum value of $y = a^{a^x} + \frac{a}{a^{a^x}}$ is $(a > 0, a, x \in R)$

(1) $2\sqrt{a}$ (2) $\sqrt{2}\,a$ (3) $2\sqrt{2}\,a$ (4) $2\sqrt{2a}$

Ans. (1)

Sol. A.M $\geq$ G.M $\Rightarrow \frac{a^{a^x} + \frac{a}{a^{a^x}}}{2} \geq \left(a^{a^x} \times \frac{a}{a^{a^x}}\right)^{1/2}$

$a^{a^x} + \frac{a}{a^{a^x}} \geq 2\sqrt{a}$

$\therefore$ Minimum value $= 2\sqrt{a}$

11. A hyperbola passes through the focii of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the product of their eccentricities is 1. Equation of hyperbola is

(1) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (3) $\frac{x^2}{9} - \frac{y^2}{5} = 1$ (4) $\frac{x^2}{5} - \frac{y^2}{9} = 1$

Ans. (2)

Sol. $\frac{x^2}{25}+\frac{y^2}{16}=1$

a = 5, b = 4

$e = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

focii : (3, 0), (–3, 0)

let equation of hyperbola is $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

satisfy (±3, 0) $\Rightarrow \frac{9}{A^2} = 1 \Rightarrow A^2 = 9$

eccentricity of hyperbola = $\frac{1}{\text{eccentricity of ellipse}} = \frac{5}{3}$

$\Rightarrow \frac{5}{3} = \sqrt{1+\frac{B^2}{9}} \Rightarrow 1 + \frac{B^2}{9} = \frac{25}{9}$

$\Rightarrow B^2 = 16$

equation of hyperbola is

$\frac{x^2}{9}-\frac{y^2}{16}=1$

12. The contrapositive of the statement :

"If you work hard then you will earn" is :

(1) If you don't work hard you will earn

(2) If you will not earn then you not work hard .

(3) you will work hard and you will not earn.

(4) Either you will work hard or you will earn

Ans. (2)

Sol. p : you work ward

q : you will earn

given $(p \rightarrow q)$

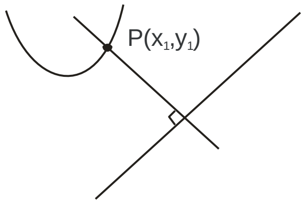
contrapositive of $(p \rightarrow q) = \sim q \rightarrow \sim p$

13. Minimum distance between the curves $x^2 = 2y$ and $y = x - 1$ is :

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Ans. (1)

Sol. $\left.\frac{dy}{dx}\right|_P = 1$



$\therefore\ x_1 = 1$

$\Rightarrow P = \left(1, \frac{1}{2}\right)$

$\therefore\ d_{min} = \left|\frac{1-1-\frac{1}{2}}{\sqrt{2}}\right| = \frac{1}{2\sqrt{2}}$

14. $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$, then $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in

(1) A.P. (2) G.P. (3) H.P. (4) none of these

Ans. (3)

Sol. $I_n = \int_{\pi/4}^{\pi/2} (\cot x)^n\, dx$

$I_n + I_{n+2} = \int_{\pi/4}^{\pi/2} ((\cot x)^n + (\cot x)^{n+2})dx = \int_{\pi/4}^{\pi/2} (\cot x)^n \operatorname{cosec}^2 x\, dx$

$\cot x = t$

$= -\int_1^0 t^n dx = \int_0^1 t^n dx = \left.\frac{t^{n+1}}{n+1}\right|_0^1 = \frac{1}{n+1}$

$\therefore\ I_n + I_{n+2} = \frac{1}{n+1}$

$\therefore\ I_2 + I_4 = \frac{1}{3}$

$I_3 + I_5 = \frac{1}{4}$

$I_4 + I_6 = \frac{1}{5}$

$\Rightarrow\ I_2 + I_4,\ I_3 + I_5,\ I_4 + I_6$ are in H.P.

15. If $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, $0 < x, y < \pi$, find the value of sin x + cosy.

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1+\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

Ans. (3)

Sol. $x = y = \frac{\pi}{3}$ satisfy the equation

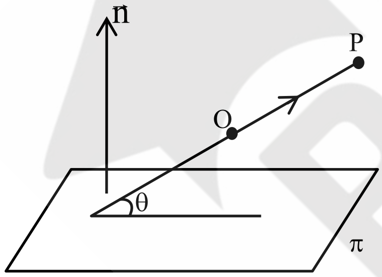
$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

16. Find the projection of $\overrightarrow{OP}$ on the plane passes through the point A(2, 1, 3), B(3, 2, 1), C(2, 4, 2); where P is the point (2, –1, 0)

(1) $\sqrt{\frac{44}{35}}$ (2) $\sqrt{\frac{47}{35}}$ (3) $\sqrt{\frac{33}{47}}$ (4) $\sqrt{\frac{41}{36}}$

Ans. (1)

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ -1 & 2 & 1 \end{vmatrix} = 5\hat{i}+\hat{j}+3\hat{k}$



$\therefore$ Required plane is : $5(x-2) + (y-1) + 3(z-3) = 0$

i.e. $5x + y + 3z = 20$

$|\overrightarrow{OP}| = \sqrt{4+1+0} = \sqrt{5}$

$|\overrightarrow{OP}| = 2\hat{i}-\hat{j}$

$\sin\theta = \left|\frac{10-1}{\sqrt{5}\sqrt{25+1+9}}\right| = \frac{9}{5\sqrt{7}}$

$\therefore$ Projection $= \sqrt{5} \times \cos\theta = \sqrt{5} \times \frac{1}{5}\sqrt{\frac{44}{7}} = \sqrt{\frac{44}{35}}$

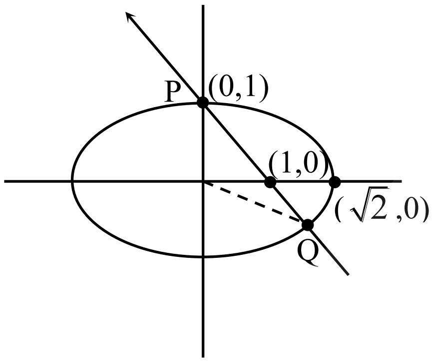
17. The line x + y = 1 cuts the ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ in points P & Q. Find the angle subtended by segment PQ at the centre of ellipse.

(1) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$ (2) $\tan^{-1}\left(\frac{1}{4}\right)$

(3) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$ (4) can't be found

Ans. (1)

Sol.



Homogenise Ellipse w.r.t. line, $\frac{x^2}{2}+\frac{y^2}{1}=(x+y)^2$

$\therefore\ x^2 + 2y^2 = 2x^2 + 2y^2 + 4xy$

$\Rightarrow x^2 + 4xy = 0$

$\Rightarrow x = 0\ ,\ y = -\frac{x}{4}$

angle between these line is $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

18. If a curve y(x) satisfies the differential equation $(2xy^2 - y)\,dx + xdy = 0$ and it passes through the point of intersection of the line x + y = 4 and 2x – 3y = –2, then find the value of |y(1)|.

Ans. 0.5

Sol. $(2xy^2 - y)\,dx = -x\,dy$

$x\,\frac{dy}{dx} = y - 2xy^2$

$\frac{dy}{dx} = \frac{1}{x}\,.\,y - 2xy^2$

$\frac{dy}{dx} - \frac{1}{x}\,.\,y = -\,2xy^2$

$y^{-2}\,\frac{dy}{dx} - \frac{1}{x}\,y^{-1} = -2$

$y^{-1} = t \Rightarrow\ -y^{-2}\,\frac{dy}{dx} = \frac{dt}{dx}$

$$-\frac{dt}{dx} - \frac{1}{x}\ t = -2$$

$$\Rightarrow \frac{dt}{dx} + \frac{1}{x}\ .\ t = 2 \qquad \text{I.F.} = e^{\ell nx} = x$$

$$tx = 2\times\frac{x^2}{2} + c \qquad \Rightarrow \frac{x}{y} = x^2 + c$$

It passes through P(2, 2)

$\therefore\ c = -3$

$\therefore\ \frac{x}{y} = x^2 - 3$

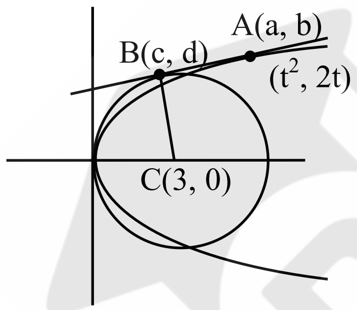
If x = 1, $\frac{1}{y} = -2 \qquad \Rightarrow y = -\frac{1}{2}$

$\therefore\ |y(1)| = \frac{1}{2} = 0.5$

19. Let the curve $(x - 3)^2 + y^2 = 9$ and $y^2 = 4x$ have common tangent touches the curves at (a, b) and (c, d) in I[st] quadrant, then find 2(a + c)

Ans. 9

Sol. Equation of tangent of A



$ty = x + t^2$

$x - yt + t^2 = 0$

$$\left|\frac{3-0+t^2}{\sqrt{1+t^2}}\right| = 3$$

$(3 + t^2)^2 = 9\ (1 + t^2)$

$t = 0, \pm\sqrt{3}$

Point A $(3, 2\sqrt{3})$ in first quadrant

For point B foot of perpendicular from c to tangent

$$\frac{x-3}{1} = \frac{y-0}{-\sqrt{3}} = -\frac{(3-0+3)}{4} \quad \Rightarrow x = \frac{3}{2}$$

$c = \frac{3}{2}$ and a = 3

2(a + c) = 9

20. If 'x' is a number divided by '4' leaves the remainder '3' then find the remainder if $(2020 + x)^{2022}$ is divided by '8'.

Ans. 1

Sol. $x = 4k + 3$; $k \in W$

$\therefore (2020 + 4k + 3)^{2022} = (8\lambda + 1)^{1011}$

$\therefore (8\lambda + 1)^{1011} = {}^{1011}C_0 + \underbrace{{}^{1011}C_1(8\lambda) + \ldots\ldots\ldots}_{\text{multiple of 8.}}$

$\therefore$ Remainder on dividing by 8 is 1

21. If $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$ and $AA^T = I$ find $\alpha^4 + \beta^4$

Ans. 1

Sol. $AA^T = I$

$\Rightarrow \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 1 + \alpha^2 = 1 \Rightarrow \alpha = 0$

$\alpha^2 + \beta^2 = 1 \Rightarrow \beta^2 = 1$

$\therefore \alpha^4 + \beta^4 = 0 + 1 = 1$

22. Find number of all two digit numbers 'n' such that $3^n + 7^n$ is divisible by 10.

Ans. 45

Sol. n is odd number

Hence n = {11, 13, 15, ….., 99}

Number of values of 'n' is 45

23. Find the value of : $\int_{-2}^{2} |(x-2)(x+1)|\, dx$

Ans. 6.33

Sol. $I = \int_0^2 |(x-2)(x+1)| + |(-x-2)(-x+1)|\, dx$

$= \int_0^2 |((-x^2+x+2)+(x+2)|x-1|)\, dx$

$= \left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Bigg|_0^2 + \int_0^1 (x+2)(-x+1)\, dx + \int_1^2 (x^2+x-2)\, dx$

$$= \left(-\frac{8}{3}+2+4\right) + \int_0^1 (-x^2-x+2)\,dx + \left(\frac{x^3}{3}+\frac{x^2}{2}-2x\right)\Bigg|_1^2$$

$$= \frac{10}{3} + \left(-\frac{x^3}{3}-\frac{x^2}{2}+2x\right)\Bigg|_0^1 + \left(\frac{8}{3}+2-4\right) - \left(\frac{1}{3}+\frac{1}{2}-2\right)$$

$$= \frac{10}{3} + \left(-\frac{1}{3}-\frac{1}{2}+2\right) + \frac{2}{3} + \frac{7}{6} = \frac{19}{3}$$

24. Out of '400' people 160 are nonveg and smoker, 140 are veg and smoker and 100 are veg and non smokers. The percentage of people suffering from a certain chest disease in 35%, 20% and 10% respectively. If a person suffers from this disease then find the probability that he is non veg and smoker.

Ans. 0.5957

Sol. Nonveg + smoker $\frac{160}{400} \xrightarrow{\text{disease}} \frac{160}{400}\times\frac{35}{100}$

Veg + smoker $\frac{140}{400} \xrightarrow{\text{disease}} \frac{140}{400}\times\frac{20}{100}$

Veg + smoker $\frac{100}{400} \xrightarrow{\text{disease}} \frac{100}{400}\times\frac{10}{100}$

Required probability = $\dfrac{\frac{160\times35}{400\times100}}{\frac{160\times35}{400\times100}+\frac{140\times20}{400\times100}+\frac{100\times10}{400\times100}}$

$= \frac{16\times35}{16\times35+14\times20+100} = \frac{560}{940} = \frac{56}{94} = \frac{28}{47} = 0.5957$

25. If $\bar{a} = 3\hat{i}+\alpha\hat{j}+k,\ \bar{b} = \hat{i}-\alpha\hat{j}+3\hat{k}$ and area of parallelogram made by $\bar{a}$ and $\bar{b}$ are adjacent sides is $8\sqrt{3}$ then $\bar{a}.\bar{b}$ is :

Ans. 2

Sol. $\bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \alpha & 1 \\ 1 & -\alpha & 3 \end{vmatrix} = 4\alpha\hat{i}-8\hat{j}-4\alpha\hat{k}$

area = $|\bar{a}\times\bar{b}| = 8\sqrt{3}$

$= \sqrt{16\alpha^2+16\alpha^2+64} = 8\sqrt{3}$

$32\alpha^2 + 64 = 64.3$

$\alpha^2 + 2 = 2.3 = 6 \Rightarrow a^2 = 4$

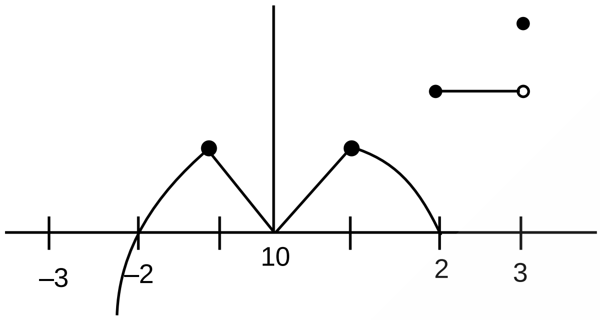
$\alpha = \pm 2$

$\bar{a}.\bar{b} = 3-\alpha^2+3 = 6-4 = 2$

26. If f(x) = $\begin{cases} \min\{|x|, 4-x^2\}, & -3 \le x \le 2 \\ [|x|] \quad , & 2 < x \le 3 \end{cases}$ then number of points of non differentiability in [–3,3]

Ans. 5

Sol. Using graph of f(x)



5 point