JEE MAIN 2020

1st Sep 2020 - 2nd Shift

Questions and Solutions by Careers360

SUBJECT: MATHEMATICS

TOTAL QUESTIIONS: 25

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Q1. Let

p: I have passed JEE (MAIN)

q: My JEE (Main) rank is 100, and

r: I will appear in JEE (Advanced)

Be three statements with truth values T, F, and T respectively. Then which one of the following has the truth value, F?

1) $(\sim p \vee \sim q) \wedge r$

2) $(\sim p \ \vee q) \wedge (\sim r)$

3) $(p \wedge \sim q) \vee (\sim r)$

4) $(\sim p \ \wedge q) \vee (r)$

Solution:

$(\ \vee q) \wedge (\sim r) = F$
$p \rightarrow T$
$q \rightarrow F$
$r \rightarrow T$

Q2. Let $A = \{1, 2, 3\}$. Define a relation R on A as

$aRb \Leftrightarrow |a^2 - b^2| \leq 5$

Then which of the following is not true?

1) R is symmetric & transitive

2) R is reflexive & symmetric

3) Range of R is A

4) $R^{-1} = R$, where R^{-1} is the inverse of R.

Solution: $aRb \Leftrightarrow |a^2 - b^2| \leq 5$

If $a = 1 \;\Rightarrow\; b = 1, \;\; b = 2$ and $b \neq 3$

$aRb = (1, 2), (1, 1)$

If $a = 2 \;\Rightarrow\; b = 1, \;\; b = 2$ and $b = 3$

$aRb = (2, 1), (2, 2), (2, , 3)$

If $a = 3 \Rightarrow b \neq 1, b = 2, b = 3$

So, $(3, 2), (3, 3)$

Option 4.

Q3. The area (in sq. units) of the region bounded by parabola $x = 2y^2$ and line $x + y = 1$ is:

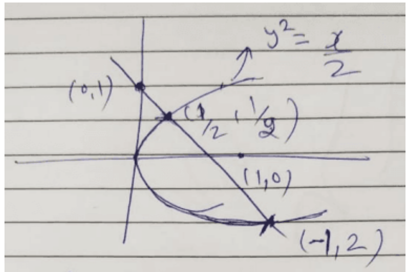
1) $\frac{9}{8}$

2) $\frac{5}{4}$

3) $\frac{13}{24}$

4) $\frac{11}{12}$

Solution:



$$y^2 = \frac{x}{2} \quad x + y = 1$$

$$y^2 = \frac{1-y}{2}$$

$$2y^2 + y - 1 = 0$$

$$y = \frac{-1 \pm \sqrt{1 + 4 \times 2 \times -1}}{2 \times 2}$$

$$y = \frac{-1 \pm 3}{4}$$

$$y = \frac{1}{2}, -1$$

$$Area = \int_{-1}^{\frac{1}{2}} (x_1 - x_2) dy$$

$$= \int_{-1}^{\frac{1}{2}} (2y^2 - 1 + y) dy$$

$$= \left[2 \times \frac{y^3}{3} - y + \frac{y^2}{2}\right]_{-1}^{\frac{1}{2}}$$

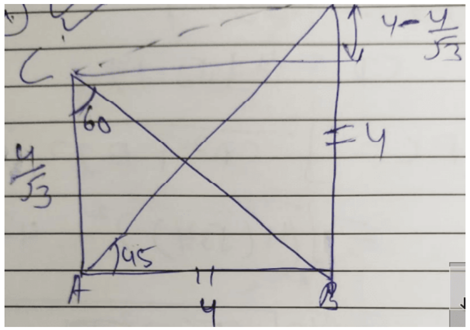
$$= \left[2 \times \frac{1}{24} - \frac{1}{2} + \frac{1}{8}\right] - \left[\frac{-2}{3} + 1 + \frac{1}{2}\right]$$

$$= \frac{9}{8}$$

Option A

Q4. The top of two poles, standing 4 m apart on a horizontal plane, are connected by a cable. If the angle of depression from the top of one pole to the bottom of another pole are 30^{o} and 45^{o}, then the minimum length (in m) of the cable is:

1) $\sqrt{\frac{32\sqrt{3} - 16}{3}}$ 2) $\sqrt{\frac{128 - 32\sqrt{3}}{3}}$ 3) $\sqrt{\frac{112 - 32\sqrt{3}}{3}}$ 4) $\sqrt{\frac{122 - 32\sqrt{3}}{3}}$

Solution. 

$$\tan 60 = \frac{AB}{AC} = \frac{4}{AC}$$

$$AC = \frac{4}{\sqrt{3}}$$

$$Cable = \sqrt{4^2 + 4^2\left(1 - \frac{1}{3}\right)^2}$$

$$= \sqrt{16 + 16\left(1 + \frac{1}{3} - \frac{2}{\sqrt{3}}\right)}$$

$$= \sqrt{16 + 16\left(\frac{4 - 2\sqrt{3}}{3}\right)}$$

$$= \sqrt{\frac{48 + 16(4 - 2\sqrt{3})}{3}}$$

$$= \sqrt{\frac{112 - 32\sqrt{3}}{3}}$$

Option C

Q5. The locus of the point

$z = x + iy \quad (x, y \in R, i = \sqrt{-1})$ satisfying $\arg\left[\frac{z-3}{z+3}\right] = \frac{\pi}{3}$ is

1) a circle with center at $(\sqrt{3}, 0)$

2) a circle with radius $= 2\sqrt{3}$

3) an ellipse with a length of its minor axis = 3

4) a circle with centre at $(0, 2\sqrt{3})$ & radius 3.

Solution: $\frac{z-3}{z+3}=\sqrt{3}$

$\frac{a+ib-3}{a+ib+3}$

$\frac{(a-3)+ib}{(a+3)+ib}\times\frac{(a-3)-ib}{(a+3)-ib}$

$\frac{(a-3)(a+3)+b^2-b(a-3)+b(a+3)}{(a+3)^2+b^2}$

$a^2-9+b^2 \quad -ba+3b+ba+3b$

$\frac{6b}{a^2-9+b^2}=\sqrt{3}$

$a^2+b^2-2\sqrt{3}b=9+3$

$a^2+(b-\sqrt{3})^2=12=(2\sqrt{3})^2$

Q6. For non-zero numbers a, b if $y(x)=\frac{a+bx^{\frac{3}{2}}}{x^{\frac{5}{4}}}, x>0$ and $y''(5)=0$ then the ratio $a:b$ is:

1) $3:\sqrt{5}$ 2) 3:1 3) $\sqrt{5}:\sqrt{3}$ 4) $\sqrt{5}:3$

Solution: $\frac{a}{x^{\frac{5}{4}}}+bx^{\frac{1}{4}}$

$\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{-5}{4}ax^{\frac{-9}{4}}+\frac{1}{4}bx^{\frac{-3}{4}}$

$\frac{\mathrm{d^2y}}{\mathrm{dx^2}}=\frac{+45ax^{\frac{-13}{4}}}{16}-b^{\frac{-7}{4}}\frac{3}{16\backslash}$

$ax^{\frac{-6}{4}}=b$

$$\frac{a}{b} = \frac{3 \times 5\sqrt{5}}{45}$$

$$\frac{a}{b} = \frac{\sqrt{5}}{3}$$

Q7. Let r and s respectively the remainders when $(12)^{23}$ and $(23)^{12}$ are divided by 5, then $r + s$ is equal to:

1) 4 2) 3 3) 5 4) 2

Solution:

$$\begin{aligned}(12)^{23} &= 12 \cdot (12^2)^{11} \\ &= \frac{12(145-1)^{11}}{5} \\ &= \frac{12 \times (-1)}{5} \\ &= -2 \\ &= 5 - 2 = 3\end{aligned}$$

$$\begin{aligned}(23)^{12} &= (529)^6 \\ &= (530-1)^6 \\ &= \frac{1}{5}\end{aligned}$$

$$\Rightarrow 3 + 1 = 4$$

Q8. if plane $P : 2x - y + 3z + 5 = 0$ is rotated through 90^o about its line intersection with the plane $5x - 4y - 2z + 1 = 0$, then the equation of the plane P in the new portion is:

1) $27x - 24y - 26z - 13 = 0$

2) $31x + 2y - 20z - 13 = 0$

3) $27x + 27y - 26z + 13 = 0$

4) $31x - 25y + 29z + 13 = 0$

Solution: $2x - y + 3z + 5 + \lambda(5x - 4y - 2z + 1) = 0$

or $(2 + 5\lambda)x - (1 + 4\lambda)y + (3 - 2\lambda)z + 5 + \lambda = 0$

This will be perpendicular to the plane $2x - y + 3z + 5 = 0$

If $2(2 + 5\lambda) + (1 + 4\lambda)3(3 - 2\lambda) = 0$

$\Rightarrow \lambda = \frac{-7}{4}$ and the required equation of the plane is

$$4(2x - y + 3x + 5) - (5x - 4y - 2z + 1) = 0$$
$$\Rightarrow 27x - 24y - 26z - 13 = 0$$

Q9. If tangent at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ to the circle $x^2 + y^2 = 1$ is also a tangent to the circle $(x - a)^2 + (y - b)^2 = 1$, then the order pair (a,b) can be :

1) $\left(-\sqrt{2}, \sqrt{2}\right)$

2) $\left(1, -2\sqrt{2}\right)$

3) $\left(-2, \sqrt{2}\right)$

4) $\left(-\sqrt{2}, 2\sqrt{2}\right)$

Solution :

Tangent to $x^2+y^2=1$ at $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$:

$$\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=1$$

$\because$ It is a tangent to $(x-a)^2-(y-b)^2=1$

$$\therefore \frac{\left|\frac{a}{\sqrt{2}}+\frac{b}{\sqrt{2}}-1\right|}{\sqrt{\frac{1}{2}+\frac{1}{2}}}=1$$

$$\therefore \left|a+b-\sqrt{2}\right|=\sqrt{2}$$

$$\therefore (a,b) \text{ can be } \left(-\sqrt{2},\sqrt{2}\right)$$

Correct option is (1)

Q10. Let A,B & C be the three events such that the probability of occurence of exactly one of A and B is $\frac{1}{2}$, the probability occurence of exactly one of B and C is $\frac{3}{4}$ and the probability occurence of exactly one of C and A is $\frac{1}{4}$. If the probability of the occurence of all the three events simultaneously is $\frac{1}{16}$, then the probability that atleast one of the three events will occur is :

1) $\frac{7}{8}$

2) $\frac{15}{16}$

3) $\frac{13}{16}$

4) $\frac{7}{16}$

Solution :

$$P(A) + P(B) - 2P(A \cap B) = \frac{1}{2}$$

$$P(B) + P(C) - 2P(B \cap C) = \frac{3}{4}$$

$$P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$$

$$P(A \cap B \cap C) = \frac{1}{16}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{3}{4} + \frac{1}{4}\right) + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$$

∴ Option (1) is correct.

Q11. The function $f(x) = \sin 2x + 2\cos 2x, x \in \left(\frac{-3\pi}{4}, \frac{3\pi}{4}\right)$ has :

1) A point of local maxima and a point of inflection.

2) A point of local maxima and a point of local minima.

3) A point of local minima and a point of inflection.

4) No critical point.

Solution :

$$f(x) = \sin 2x + 2\cos 2x$$

$$f'(x) = 2\cos 2x - 2\sin x$$

$$= 2(1 - 2\sin^2 x) - 2\sin x$$

$$= 2\left[-2\sin^2 x - \sin x + 1\right]$$

$$= -2\left[2\sin^2 x + \sin x - 1\right]$$

$$= -2(\sin x + 1)(2\sin x - 1)$$

$$f'(x) = 0 \text{ at } x = \frac{-\pi}{2} \text{ and } \frac{\pi}{2}$$

$$f''(x) = -4\sin 2x - 2\cos x$$

$$f''(x) = 0 \text{ at } x = \frac{-\pi}{2} \text{ (point of inflection)}$$

$$f''(x) = 0 \text{ negative at } x = \frac{\pi}{6} \text{ (maxima)}$$

Correct option is (1).

Q12. If the system of equations

$$2x + y + pz = -1$$

$$3x - 2y + z = q$$

$$5x - 8y + 9z = 5$$

has more than one solution, then $q - p$ is equal to :

1) -4

2) 4

3) -2

4) 2

Solution :

$$\Delta=\begin{vmatrix}2 & 1 & p\\ 3 & -2 & 1\\ 5 & -8 & 9\end{vmatrix}=0$$

$$\Rightarrow 2(-10)-1(22)+p(-14)=0$$

$$\Rightarrow 14p=-20-22$$

$$\Rightarrow p=-3$$

$$\Delta_1=\begin{vmatrix}-1 & 1 & -3\\ q & -2 & 1\\ 5 & -8 & 9\end{vmatrix}=0$$

$$\Rightarrow -1(-10)-q(-15)+5(-5)=0$$

$$\Rightarrow 15q=15$$

$$\Rightarrow q=1$$

$$\therefore q-p=4$$

Correct option is (2)

Q13. If both the roots of the equation $x^2-2ax+a^2-1>0\ \ (a\in\mathbb{R})$ lie in the interval $(-2,2)$, then the equation $x^2-(a^2+1)x-(a^2+2)=0$ has

1) one root in $(-3,-2)$ and another root in $(0,2]$

2) one root in $(0,2)$ and another root in $(-2,0)$

3) one root in $[2,3)$ and another root in $(-2,0)$

4) both roots in $(-3,0)$

Solution :

$x^2-2ax+a^2-1=0$

$-2<\frac{2a}{2}<2 \Rightarrow -1<a<1$

at $x=-2$

$4+4a+a^2-1>0$

$a^2+4a+3>0$

$\Rightarrow a \in(-\infty,-3) \cup(-1, \infty)$

at $x=2$

$4-4a+a^2-1>0$

$a^2-4a+3>0$

$\Rightarrow a \in(-\infty, 1) \cup(3, \infty)$

$\therefore a \in(-1,1)$

$f(x)=x^2-(a^2+1)x-(a^2+2)$

$f(0)=-(a^2+2)<0$

$f(2)=4-2(a^2+1)-(a^2+2)<0$

$$f(2) = 4 + 2(a^2+1) - (a^2+2) > 0$$

∴ Correct option is (3)

Q14. The integral $\int \frac{\cos^5\theta}{\sin\theta(4-3\sin^2 2\theta)}\mathrm{d}\theta$ is equal to : (where c is constant of Integration)

1) $\frac{1}{24}\log_e\left(\frac{\sin^6\theta}{\sin^6\theta+\cos^6\theta}\right)+c$

2) $\frac{1}{5}\log_e\left|\frac{\cos^5\theta}{\sin^5\theta+\cos^5\theta}\right|+c$

3) $\frac{1}{6}\log_e\left(\frac{\cos^6\theta}{\sin^6\theta+\cos^6\theta}\right)+c$

4) $\frac{1}{20}\log_e\left|\frac{\sin^5\theta}{\sin^5\theta+\cos^5\theta}\right|+c$

Solution :

$$\int \frac{\cos^5\theta}{\sin\theta(4-3\sin^2 2\theta)}\mathrm{d}\theta$$

$$= \int \frac{\cos^5\theta}{\sin\theta(4-3\times(2\sin\theta\cos\theta)^2)}\mathrm{d}\theta$$

$$= \frac{1}{4}\int \frac{\cos^5\theta}{\sin\theta(1-3\sin^2\theta\cos^2\theta)}\mathrm{d}\theta$$

$$= \frac{1}{4}\int \frac{\mathrm{d}\theta}{\tan\theta(\sec^4\theta-3\tan^2\theta)}$$

$$= \frac{1}{4}\int \frac{\sec^4\theta d\theta}{\tan\theta(1+\tan^2\theta)((1+\tan^2\theta)^2-3\tan^2\theta)}$$

Let $\tan\theta = t$

$\sec^2\theta \ d\theta = t \ dt$

$$= \frac{1}{4}\int \frac{dt}{t(1+t^2)((1+t^2)^2-3t^2)}$$

$$= \frac{1}{4}\int \frac{dt}{t(1+t^2)(t^4-t^2+1)}$$

$$= \frac{1}{4}\int \frac{t dt}{(t^4+t)(t^4-t^2+1)}$$

$t^2 = u \Rightarrow 2t \ dt = du$

$$= \frac{1}{8}\int \frac{du}{(u^2+u)(u^2-u+1)}$$

$$= \frac{1}{8}\int \frac{du}{u(u+1)(u^2-u+1)}$$

$$\frac{1}{u(u+1)(u^2-u+1)} = \frac{A}{u} + \frac{B}{u+1} + \frac{Cu+D}{u^2-u+1}$$

$$\therefore A = 1, B = \frac{-1}{3}, C = \frac{-2}{3}, D = \frac{1}{3}$$

$$\frac{1}{8}\left[\frac{1}{3}\left\{3\ln u - \ln|u+1| - \ln|u^2-u+1\right\}\right] + c$$

$$\frac{1}{24}\left\{\ln\left|\frac{u^3}{(u+1)(u^2-u+1)}\right|\right\}+c$$

$$\frac{1}{24}\left\{\ln\left|\frac{u^3}{u^3+1}\right|\right\}+c$$

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15) The sum

$\frac{-2}{3}+\frac{2}{9^2}+\frac{10}{9^3}+\frac{18}{9^4}+\frac{26}{9^5}+\cdots$ upto infinity is equal to

$1)\frac{17}{24}\quad 2)\frac{-11}{24}\quad 3)\frac{-5}{8}\quad 4)\frac{5}{8}$

Solution: $S_n=\frac{6}{9}+\frac{2}{9^2}+\frac{10}{9^3}\cdots$

$$\frac{S_n}{9}=\frac{-6}{9^2}+\frac{2}{9^3}+\cdots$$

$$\frac{8S_n}{9}=\frac{-6}{9}+\frac{8}{9^2}+\frac{18}{9^3}\cdots$$

$$=\frac{-6}{9}+\frac{\frac{8}{9^2}}{1-\frac{1}{9}}$$

$$=\frac{-6}{9}+\frac{8}{9^2\times\frac{8}{9}}$$

$$= \frac{-6}{9} + \frac{1}{9}$$

$$\frac{8S_n}{9} = \frac{-5}{9} \Rightarrow S_n = \frac{-5}{8}$$

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16. Let $y = y(x)$ be the equation of the differential equation

$x \log_e x \frac{dy}{dx} + 2y = 2x (x > 1)$ if $\lim_{x \to 1^+} y(x) = 1, \ then \ y(e^x)$ is equal to:

1) $\frac{e^2 - 1}{2}$ 2) $e^2 + 1$ $\frac{e^2 + 1}{2}$ 3) $e^2 - 1$

Solution:

$$x \ln x \frac{dy}{dx} + 2y = 2x$$

$$\frac{dy}{dx} + 2y \cdot \frac{1}{x \ln x} = \frac{2x}{x \ln x}$$

$$Linear \ \ D.2.$$

$$I.F := e^{\int \frac{2}{x \ln x} dx}$$

$$\ln x = t$$

$$\frac{1}{x} dx = dt$$

$$I.f. = e^{t^2} = (\ln x)^2$$

$$y.If = \int \frac{2}{\ln x} \cdot (\ln x)^2 dx$$

$$y(\ln x)^2 = 2 \int \ln x dx$$

$$y(\ln x)^2 = 2(x\ln x - x) + C$$
$$x = 1 \quad y = 1$$
$$C = 2$$

Now at

$$x = e^2$$
$$y.4 = 2(e^2.2 - e^2) + 2$$
$$y.4 = 2C^2 + 2$$
$$y = \frac{e^2+1}{2}$$

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17. Let f be a differentiable function satisfying $f(x+y) = f(x) + f(y) - xy$ for all $x, y \equiv R$. If $\lim_{h\to 0}\frac{f(h)}{h} = 3,\ then\ \sum_{n=1}^{10} f(n)$ is equal to:

$1)\frac{-55}{2} \quad 2)\frac{275}{2} \quad 3)\frac{225}{4} \quad 4)\frac{-55}{4}$

Solution :

$$f(x+h) = f(x) + f(14) - xy$$

$$\lim_{h\to 0}\frac{f(h)}{h} = 3$$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x) + f(h) - xh - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(h) - xh}{h}$$

$$= \lim_{h \to 0} \frac{f(h)}{h} - x = 3 - x$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - x$$

$$\int dy = \int (3 - x) dx$$

$$\Rightarrow y = 3x - \frac{x^2}{2} + c$$

$$\Rightarrow y = 3x - \frac{x^2}{2} (c = 0 \ \ as \ \ at \ \ x = 0, y = 0)$$

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18) All the words (with or without meaning) for med using all the five letters of the words GOING are arranged as in dictionary then the word OGGIN occur at the place which is :

$1)99^{th} \quad 2)51^{th} \quad 3)48^{th} \quad 4)50^{th}$

Solution:

$GOING \rightarrow$

$GGINO \rightarrow 4!$

$IGGNO \rightarrow \frac{4!}{2!}$

$NGGIO \rightarrow \frac{4!}{2!}$

$OGGIN \rightarrow$

$4! + 4! = 1 = 49^{th}$

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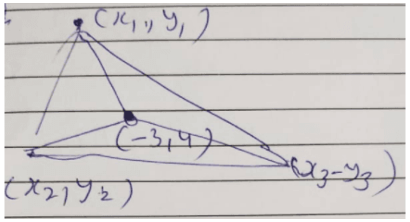
19. If $(x_i, y_i), \quad i = 1, 2, 3$ are the vertices of an equivalent triangle such that

$(x_1+3)^2+(y_1-4)^2=(x_2+3)^2+(y_2-4)^2=(x_3+3)^2+(y_3-4)^2$, then tha value of $\frac{y_1+y_2+y_3}{x_1+x_2+x_3}$ is :

$1)\frac{-4}{3} \quad 2)\frac{4}{3} \quad 3)\frac{-3}{4} \quad 4)\frac{3}{4}$

Solution:



$$\frac{x_1+x_2+x_3}{3}=-3$$

$$\frac{y_1+y_2+y_3}{3}=4$$

$$x_1+x_2+x_3=-9$$

$$y_1+y_2+y_3=12$$

$$\frac{y_1+y_2+y_3}{x_1+x_2+x_3}=\frac{12}{(-9)}=\frac{-9}{3}$$

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20. The foci of a hyperbole coincide with the foci of the ellipse, $\frac{x^2}{16}+\frac{y^2}{9}=1$ if the eccentricity of hyperbole is 4, then its equation is :

1) $\frac{x^2}{7} - \frac{y^2}{105} = \frac{1}{16}$

2) $\frac{x^2}{7} - \frac{4y^2}{105} = \frac{1}{4}$

3) $\frac{x^2}{20} - \frac{y^2}{75} = \frac{1}{16}$

3) $\frac{x^2}{5} - \frac{y^2}{75} = \frac{1}{16}$

Solution : Equation of ellipse = $\frac{x^2}{16} + \frac{y^2}{9} = 1$

eccentricity of ellipse $\Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}}$

$a^2 = 16, b^2 = 9 \qquad e = \frac{\sqrt{7}}{4}$

$(ae, 0) = \left(\sqrt{7}, 0\right)$

This is also focus of hyper bola ,

For hyperbola,

$(4a, 0) = (\sqrt{7}, 0)$

$a = \frac{\sqrt{7}}{4}$

eccentricity of hyperbola = $4 = \sqrt{1 + \frac{b^2}{a^2}}$

$$16 = 1 + \frac{b^2}{7} \times 16$$

$$b = \frac{\sqrt{105}}{4}$$

For Hyperbola we get

$$a = \frac{\sqrt{7}}{4}, \quad b = \frac{\sqrt{105}}{4}$$

equation of hyperbola

$$\frac{x^2}{\frac{7}{16}} - \frac{y^2}{\frac{105}{16}} = 1$$ or $$\frac{x^2}{7} - \frac{y^2}{105} = \frac{1}{16}$$

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21. For $b \equiv N$, if the limit of the function

$$f(x) = \begin{cases} (5x-4), & x < 4 \\ \frac{[x+b]}{2}, & x > 4 \end{cases}$$

where $x \to 4$, exists and $[t]$ denotes the greatest integer $\leq t$, then b is equal to ______.

Solution :

$$f(x) = \begin{cases} (5x-4), & x < 4 \\ \frac{[x+b]}{2}, & x > 4 \end{cases}$$

$$\lim_{x \to 4}(5x-4) = 16 = \lim_{x \to 4^+} \frac{[x+b]}{2}$$

$$[x+b] = 32$$

$b = 28$

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22. If $S = \{x \equiv [0, 4\pi] : \sin 2x - \cos x = 0\}$ and the sum of the elements of S is $k\pi$, then k is equal to ____.

Solution :

$\sin(2x) - \cos(x) = 0$

$2\sin x cosx - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \quad or \quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{2} + 2\pi n \quad x = \frac{\pi}{6} + 2\pi n$

$\frac{3\pi}{2} + 2\pi n \qquad \frac{5\pi}{6} + 2\pi n$

Solution in $[0, 4\pi]$ Solution in $[0, 4\pi]$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \qquad \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$sum = 8\pi \qquad sum = 6\pi$

total $k\pi = 8\pi + 6\pi = 14\pi$

$k = 14$

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23. If the equation of a plane p passing through point $\hat{i} + \hat{j} + \hat{k}\ is\ \hat{r}.\left(2\hat{i} - \hat{j} + a\hat{k}\right) = 3$ then the length of the perpendicular from the point $4\hat{i} - \hat{j} + 3\hat{k}$ on the plane p is ___.

Solution : $\hat{i}+\hat{j}+\hat{k}\ is\ \hat{r}.\left(2\hat{i}-\hat{j}+a\hat{k}\right)=3$

$2-1+a=3$

$\Rightarrow a=2$

$$\therefore \vec{r}.\left(2\hat{i}-\hat{j}+2\hat{k}\right)=3$$

$\therefore 2x-y+2z=3$

$Point\equiv(4,-1,3)$

$\therefore\ length\ of\ \ perpendicular$

$=\dfrac{|2\times4-1(-1)+2\times3-3|}{\sqrt{2^2+(-1)^2+2^2}}$

$=\dfrac{|8+1+6-3|}{3}=\dfrac{12}{3}=4$

24. If the mean of numbers $-7,-5,-2,x,5\ \ is\ \ -1$, then the variance of the numbers $x-3,4x,x+3$ is _____.

Solution:

Given number

$-7,-5,x,5,-2$

$$mean=\frac{-7-5+x+5-2}{5}=-1$$

$x=4$

Then the variance of the number

$$x-3,4x,x+3$$

$put\ \ x=4$

$i.e\ \ 1,16,7$

mean of this three no. is 8

$$variance = \frac{(1-8)^2 + (16-8)^2 + (7-8)^2}{3}$$
$$= \frac{49+64+1}{3} = \frac{114}{3} = 38$$
$$variance = 38$$

25.Let $B = \begin{bmatrix} 22 & -26 \\ -6 & 10 \end{bmatrix}$ be the matrix obtained by applying the elementary row operations $R_2 \to R_2 - R_1, R_1 \to 2R_1 \ and \ R_1 \to R_1 - 2R_2$ (in this order only)on a matrix A then the sum of all entries of A is ___.

Solution : Let matrix A be

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$Now, \ \ R_2 \to R_2 - R_1$$
$$\begin{bmatrix} a & b \\ c-a & d-b \end{bmatrix}$$
$$Now, R_1 \to 2R_1 \ \ we \ \ get \ \ \begin{bmatrix} 2a & 2b \\ c-a & d-b \end{bmatrix}$$
$$again R_1 \to R_1 - 2R_2$$
$$= \begin{bmatrix} 2a-2(c-a) & 2b-2(d-b) \\ c-a & d-b \end{bmatrix}$$
$$= \begin{bmatrix} 2a-2c+2a & 2b-2d+2b \\ c-a & d-b \end{bmatrix} = \begin{bmatrix} 4a-2c & 4b-2d \\ c-a & d-b \end{bmatrix}$$

This is equal to matrix B

$$B = \begin{bmatrix} 22 & -26 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4a-2c & 4b-2d \\ c-a & d-b \end{bmatrix}$$

$c - a = -6 \quad 2a - c = 11$

$d - b = 10 \quad 2b - d = -13$

$a = 5$

$b = -3$

$c = -1$

$d = 7$

$Sum\ of\ all\ elements\ of\ A$

$A = 5 + 7 + (-3) + (-1)$

$A = 8$
