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JEE MAIN 2023

JAN ATTEMPT

PAPER-1 (B.Tech / B.E.)

QUESTIONS & SOLUTIONS

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24 JANUARY, 2023

03:00 PM to 06:00 PM

Duration : 3 Hours | Maximum Marks : 300

SUBJECT - MATHEMATICS

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MATHEMATICS

SECTION-A

1. Find number of numbers greater then 7000 which can be formed by using the digits 3, 5, 6, 7 and 8. Repetition of digits is not allowed.

(1) 68 (2) 168 (3) 120 (4) 172

Ans. **(2)**

Sol. Number of digit number

7			

$4 \times 3 \times 2 = 24$

8			

$4 \times 3 \times 2 = 24$

Number of 5 digit number

$5! = 120$

$\therefore$ Total number of numbers = 24 + 24 + 120 = 168

2. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\, dx$ is equal to-

(1) 2π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) π

Ans. **(1)**

Sol. $$\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{24}{\sqrt{\frac{9}{4}-x^2}} = 24.\sin^{-1}\frac{2x}{3}\Big|_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} = 24\left(\sin^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}\frac{1}{\sqrt{2}}\right) = 2\pi$$

3. If system of equation x + 2y = 6, x –3y + 72z = 0, x +y + λz = μ + 9 has infinite solution then ordered pair (λ, μ) is

(1) $\left(\frac{72}{5},\frac{-21}{5}\right)$ (2) $\left(\frac{21}{5},\frac{-72}{5}\right)$ (3) $\left(\frac{-21}{5},\frac{72}{5}\right)$ (4) $\left(\frac{-21}{5},\frac{-72}{5}\right)$

Ans. **(1)**

Sol. $$\begin{vmatrix} 1 & 2 & 0 \\ 1 & -3 & 72 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \qquad \Rightarrow \lambda = \frac{72}{5}$$

$$\Delta_x = \begin{vmatrix} 6 & 2 & 0 \\ 0 & -3 & 72 \\ \mu+9 & 1 & \lambda \end{vmatrix} = 0$$

solving $\mu = -\frac{21}{5}$

4. Consider a 3 × 3 matrix P such that $|\text{adj (adj (adj P))}| = (12)^4$, then find $|P^{-1} \cdot \text{adj } P|$

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{\sqrt{3}}$

Ans. (1)

Sol. $|P|^{2^3} = 12^4 \Rightarrow |P|^8 = 12^4 \Rightarrow |P| = 12^{\frac{1}{2}} = 2\sqrt{3}$

$|P^{-1} \text{ adj } P| = |P^{-1}|\,|\text{adj } P| = \frac{1}{|P|} \times |P|^2 = |P| = 2\sqrt{3}$

5. Let $f(x) = \frac{2^{2x}}{2^{2x}+2}$, then $\sum_{r=1}^{2022} f\left(\frac{r}{2023}\right)$ is

(1) 1010 (2) $\frac{2023}{2}$ (3) 1011 (4) $\frac{2021}{2}$

Ans. **(3)**

Sol. $f(x) = \frac{4^x}{4^x+2} \Rightarrow f(x) + f(1-x) = 1$

$\therefore \sum_{r=1}^{2022} f\left(\frac{r}{2023}\right) = \left[f\left(\frac{1}{2023}\right)+f\left(\frac{2022}{2023}\right)\right] + \left[f\left(\frac{2}{2023}\right)+f\left(\frac{2021}{2023}\right)\right] + ...$

$......+\left[f\left(\frac{1011}{2023}\right)+f\left(\frac{1012}{2023}\right)\right] = 1011$

6. If $\frac{dy}{dx} = \frac{3y^2-x^2}{3xy}$, y(1) = 1, find $6y^2(e)$

(1) e^2 (2) $\frac{e^2}{2}$ (3) $\frac{e^2}{3}$ (4) $3e^2$

Ans. (3)

Sol. $y = mx \Rightarrow \frac{dy}{dx} = m + x\frac{dm}{dx}$

$m + x\frac{dm}{dx} = \frac{3m^2x^2-x^2}{3mx^2} = \frac{3m^2-1}{3m}$

$x\frac{dm}{dx} = \frac{3m^2-1-3m^2}{3m}$

$3m\, dm = -\frac{dx}{x}$

$3\frac{m^2}{2} = -\ell n\, x + c$

$$\frac{3}{2}\frac{y^2}{x^2} = -\ell n\, x + c$$

Given x = 1, y = 1

$$\Rightarrow \quad c = \frac{3}{2}$$

$$\frac{3}{2}\frac{y^2}{x^2} = -\ell n\, x + \frac{3}{2}$$

At x = e, $\frac{3}{2}\frac{y^2}{e^2} = -1 + \frac{3}{2} = \frac{1}{2}$

$3y^2 = e^2$

$y^2(e) = \frac{e^2}{3}$

$\therefore \quad 6y^2(e) = 2e^2$

7. If $\frac{1^3 + 2^3 + 3^3 + ... + n^3}{1.3 + 2.5 + 3.7......n \text{ terms}} = \frac{9}{5}$ then the value of n is-

(1) 5 (2) 8 (3) 9 (4) 10

Ans. **(1)**

Sol. $$\frac{\left(\frac{n(n+1)}{2}\right)^2}{2.\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{2n+1}{3} + \frac{1}{2}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{3}{2}n(n+1)}{4n+2+3} = \frac{9}{5}$$

$$\Rightarrow \frac{15}{2}(n^2 + n) = 9(4n + 5)$$

$5n^2 + 5n = 24n + 30$

$5n^2 - 19n - 30 = 0$

$5n^2 - 25n + 6n - 30 = 0$

$(5n + 6)(n - 5) = 0 \Rightarrow n = 5$

8. $\left(\dfrac{1+\cos\frac{2\pi}{9}+i\sin\frac{2\pi}{9}}{1+\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}}\right)^3$ is equal to

(1) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (2) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (3) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (4) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Ans. **(1)**

Sol. $\left(\dfrac{2\cos^2\frac{\pi}{9}+2i\cos\frac{\pi}{9}\cdot\sin\frac{\pi}{9}}{2\cos^2\frac{\pi}{9}-2i\cos\frac{\pi}{9}\cdot\sin\frac{\pi}{9}}\right)^3 = e^{i\frac{2\pi}{3}} = -\frac{1}{2}+i\frac{\sqrt{3}}{2}$

9. If $f(x) = x^3 + x^2 f'(1) + xf''(2) - f'''(3)$. Then the relation between $f'(1)$, $f''(2)$, $f'''(3)$

(1) $f(0) = f'(1) + 3f''(2) + f''(3)$ (2) $f(0) = 2f'(1) + 3f''(2) - f''(3)$

(3) $f(0) = 2f'(1) - f''(2) + f'''(3)$ (4) $f(0) = 3f'(1) - f''(2) - 3f''(3)$

Ans. **(3)**

Sol. $f'(x) = 3x^2 + 2xf'(1) + f''(2) \Rightarrow f'(1) + f''(2) + 3 = 0$

$f''(x) = 6x + 2f'(1) \Rightarrow 2f'(1) - f''(2) + 12 = 0$

$f'''(x) = 6$

$\therefore f'(1) = -5$

$f''(2) = 2$

$f'''(3) = 6$

$f(0) = -6$

10. $\sim(p \wedge (p \rightarrow \sim q))$ is equivalent to-

(1) $p \rightarrow q$ (2) $p \wedge q$ (3) $p \vee q$ (4) $p \leftrightarrow q$

Ans. **(1)**

Sol. $\sim p \vee (\sim(p \rightarrow \sim q))$

$\sim p \vee (p \wedge q) = p \rightarrow q$

11. The sum of coefficients of first 3 terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n$ is 376. Find the coefficient of x^4.

(1) 695 (2) 410 (3) 405 (4) 395

Ans. (3)

Sol. ${}^nC_0 - {}^nC_1(3) + {}^nC_2(9) = 376$

$1 - 3n + \frac{9n(n-1)}{2} = 376$

$2 - 6n + 9n^2 - 9n = 752$

$9n^2 - 15n - 750 = 0$

$3n^2 - 5n - 250 = 0$

$\Rightarrow n = 10$

$T_{r+1} = {}^{10}C_r(x)^{10-r}\left(\frac{-3}{x^2}\right)^r$

$T_3 = 405$

12. If $\lim_{x\to a}$ $[x-5]-[2x+2]=0$, (where [ ] denotes greatest integer function) then 'a' belongs to

(1) $\left(-\frac{15}{2},-\frac{13}{2}\right)$ (2) $\left[-\frac{15}{2},-\frac{13}{2}\right)$ (3) $\left(-\frac{15}{2},-\frac{13}{2}\right]$ (4) $\left[-\frac{15}{2},-\frac{13}{2}\right]$

Ans. **(1)**

Sol. f(x) is continuous $\forall x \in R-\left\{n+\frac{1}{2}\right\}, n\in I$

$\therefore \quad \lim_{x\to a} f(x) = f(a)$

Hence $[a-5]-[2a+2]=0$

$\Rightarrow \quad [a]-[2a]=7$

$a \in I \; a=-7$

$a \notin I \; a = I + f$

$-I-[2f]=7$

Case-I : $f \in \left(0,\frac{1}{2}\right)$

$-I=7$

$I=-7$

$a\in(-7.5,-6.5)$

Case-II : $f\in\left(\frac{1}{2},1\right)$

$I=-8$

$\Rightarrow a\in(-7.5,-7)$

At $a=n+\frac{1}{2}, n\in I$

LHL $\neq$ RHL

$\therefore \quad a\in(-7.5,-6.5)$

SECTION-B

13. Let $a_1, a_2, \ldots\ldots, a_6$ are in Arithmetic Progression where $a_1+a_3=10$. If mean of $a_1, a_2, \ldots\ldots, a_6$ is $\frac{19}{2}$, then find the value of $8\sigma^2$ (where σ^2 denotes the variance of given numbers)

Ans. 210

Sol. $a_1, a_2, \ldots\ldots, a_6$

mean = $\frac{19}{2}$

variance = σ^2

$a_1+a_3=10$

$8\sigma^2=?$

$$\frac{a_1+a_2+a_3+a_4+a_5+a_6}{6}=\frac{19}{2}$$

$a_1+a_2+a_3+a_4+a_5+a_6=57$

$a_2 + a_4 + a_5 + a_6 = 47$

$\sigma^2 = \frac{1}{6} \sum x_i^2 - \left(\frac{19}{2}\right)^2$

$a_1 + d + a_1 + 3d + a_1 + 4d + a_1 + 5d = 47$

$4a_1 + 13d = 47$

$a_1 + a_1 + 2d = 10$

$a_1 + d = 5$

$4a_1 + 13(5 - a_1) = 47$

$a_1 = 2, d = 3$

2, 5, 8, 11, 14, 17

$\sigma^2 = \frac{1}{6}(4 + 25 + 64 + 121 + 196 + 289) - \left(\frac{19}{2}\right)^2$

$= \frac{1}{6} \times 699 - \frac{361}{4} = \frac{699}{6} - \frac{361}{4}$

$\therefore 8\sigma^2 = 210$

2. If urn 1 contain 7 red & 3 green balls, urn2 contain 3 red and 2 green balls, urn 3 contain λ red & 2 green balls. One urn is selected at random & one ball is drawn. If probability of getting red ball is 0.6 then find value of λ.

Ans. **(2)**

Sol. $\frac{1}{3}\left[\frac{7}{10}+\frac{3}{5}+\frac{\lambda}{\lambda+2}\right]=0.6 \Rightarrow \quad \cdot 7 + \cdot 6 + \frac{\lambda}{\lambda+2} = 1.8 \quad \Rightarrow \frac{\lambda}{\lambda+2} = \cdot 5 = \frac{1}{2} \Rightarrow 2\lambda = \lambda + 2$

$\lambda = 2$

3. Relation R on the set P = {a, b , c, d} is given by R = {(a, b), (b, c), (b, d)}. Find the minimum number of ordered pairs to be added in R so that it is an equivalence relation.

Ans. 13

Sol. R = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (c, a), (c, d), (d, c), (a, d), (d, a)}

minimum no. of ordered pairs = 13

4. Consider a matrix of order 5×5 which can be formed using numbers 0 or 1. How many such matrices can be formed in which sum of elements in each column & each row is 1.

Ans. 120

Sol. $\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$

I row has 5 options to place '1'

II row has 4 options

III row has 3 options

IV row has 2 options

V row has 1 options

so total matrix = $5 \times 4 \times 3 \times 2 \times 1 = 120$

5. Consider a function f(x) such that $f(x + y) = f(x) \cdot f(y)$ & $f(1) = 3$. If $\sum_{k=1}^{n} f(k) = 3279$. Find 'n'.

Ans. **7**

Sol. Put $x = y = 1$, $f(2) = 3^2$

Put $x = 2, y = 1$, $f(3) = 3^3$

and so on

$\Rightarrow f(x) = 3^x$; $x \in N$

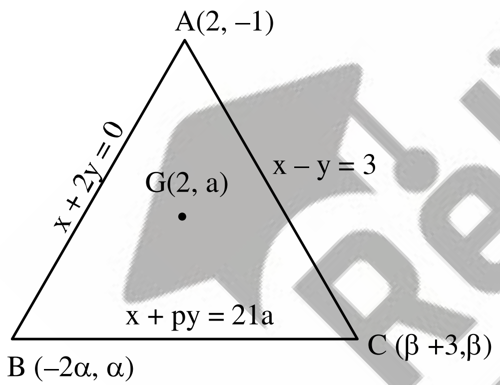
$\sum_{r=1}^{n} f(r) = 3 + 3^2 + ... + 3^n = 3279$

$\Rightarrow n = 7$

6. Consider a triangle formed by lines AC : $x - y = 3$, AB : $x + 2y = 0$ & BC : $x + py = 21a$. If centroid is (2, a), find $\ell(BC)^2$.

Ans. 17

Sol. $\frac{-2\alpha + 2 + \beta + 3}{3} = 2 \Rightarrow \beta = 1 + 2\alpha$ so C $(2\alpha + 4, 1 + 2\alpha)$



$\frac{\alpha - 1 + \beta}{3} = a \Rightarrow \alpha + \beta = 3a + 1 \Rightarrow \alpha + 2\alpha + 1 = 3\alpha + 1 = 3a + 1 \Rightarrow \alpha = a$, $\beta = 1 + 2a$

B & C lies on $x + py = 21a$

$\Rightarrow$ $-2\alpha + p\alpha = 21\alpha$ & $2\alpha + 4 + p(1 + 2\alpha) = 21a$

also $-2a + pa = 21a$ $2a + 4 + p + 2pa = 21a$

$pa = 23a$ $2a + 4 + p + 46a = 21a$

$\Rightarrow$ $a = 0$ or $p = 23$ (rejected) $p + 4 = -27a$

$p = -4$

so B(0, 0), C(4, 1)

$BC = \sqrt{16+1} = \sqrt{17}$

so $(BC)^2 = 17$

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