JEE (Main) 2020
COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 06 September, 2020 (SHIFT-2) | TIME: (03.00 p.m. to 06.00 p.m)
Duration: 3 Hours | Max. Marks: 300
SUBJECT: MATHEMATICS

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Many Dreamers... Many Achievers...
1. If \( \alpha, \beta \) are the roots of equation \( 2x (2x + 1) = 1 \) then \( \beta = ? \)
   \( (1) \beta^2 \)
   \( (2) -2x (x + 1) \)
   \( (3) 2x (x + 1) \)
   \( (4) \alpha (\alpha - 1) \)

   **Ans.**  \( (2) \)

   **Sol.**
   Given equation is \( 2x (2x + 1) = 1 \Rightarrow \) \( 4x^2 + 2x - 1 = 0 \) .......... (1)
   roots of equation (1) are \( \alpha \) and \( \beta \)
   \[ \therefore \alpha + \beta = \frac{1}{2} \Rightarrow \beta = \frac{1}{2} - \alpha \] .......... (2)
   and
   \[ 4\alpha^2 + 2\alpha - 1 = 0 \Rightarrow \alpha^2 = \frac{\frac{1}{4} - \frac{\alpha}{2}}{2} \] .......... (3)
   now
   \[ -2\alpha (\alpha + 1) = -2\alpha^2 - 2\alpha \]
   \[ = -2 \left( \frac{1}{4} - \frac{\alpha}{2} \right) - 2\alpha = -\frac{1}{2} - \alpha = \beta \]

2. A plane intersects the \( x, y, z \) axis at \( A, B, C \) respectively. If \( G(1,1,2) \) is centroid of \( \triangle ABC \), then the equation of the line perpendicular to plane and passing through \( G \) is
   \( (1) \) \[ \frac{x - 1}{1} = \frac{y - 2}{2} = \frac{z - 2}{2} \]
   \( (2) \) \[ \frac{x - 1}{2} = \frac{y - 2}{1} = \frac{z - 2}{2} \]
   \( (3) \) \[ \frac{x - 1}{2} = \frac{y - 1}{2} = \frac{z - 2}{6} \]
   \( (4) \) \[ \frac{x - 1}{3} = \frac{y - 1}{3} = \frac{z - 2}{6} \]

   **Ans.**  \( (3) \)

   **Sol.**
   Let \( A(\alpha,0,0), B(0,\beta,0), C(0,0,\gamma) \) then \( G \left( \frac{\alpha + \beta + \gamma}{3}, \frac{0 + 0 + 0}{3}, \frac{0 + 0 + 0}{3} \right) = (1,1,2) \)
3. Total number of words (with or without meaning) from letters of word 'LETTER' if no two vowels are together

(1) 100  (2) 110  (3) 120  (4) 180

Ans. (3)

Sol. Consonants are L, T, T, R
Vowels are E, E.

Total number of words (with or without meaning) from letters of word 'LETTER' = \( \frac{6!}{2!} \) = 180

Total number of words (with or without meaning) from letters of word 'LETTER' if vowels are together = \( \frac{5!}{2!} \) = 60

\( \therefore \) Required = 180 – 60 = 120

4. If the constant terms in the expansion of \( \left( \sqrt[3]{x} - \frac{K}{x^2} \right)^{10} \) is 405 then \( K = ? \)

(1) 9  (2) \( \frac{9}{2} \)  (3) 3  (4) 6

Ans. (3)

Sol. \( T_{r+1} = \binom{10}{r} \left( \frac{-K}{x^2} \right)^{10-r} \cdot \left( \sqrt[3]{x} \right)^r \)

\( = \binom{10}{r} \left( \frac{-K}{x^2} \right)^{10-r} \cdot x^{\frac{r}{3}} \)

for constant term \( \Rightarrow 5 - \frac{5r}{2} = 0 \Rightarrow r = 2 \)

\( \Rightarrow T_2 = \binom{10}{2} \cdot K^2 = 405 \)

\( \Rightarrow K^2 = \frac{405 \cdot 2}{10} \)

\( \Rightarrow K = 3 \Rightarrow K = 3 \)

5. Centre of a circle passing through point (0,1) and touching the curve \( y = x^2 \) at (2,4) is

(1) \( \left( \frac{16}{5}, \frac{53}{10} \right) \)  (2) \( \left( -\frac{16}{5}, \frac{53}{10} \right) \)  (3) \( \left( -\frac{16}{5}, -\frac{53}{10} \right) \)  (4) \( \left( \frac{16}{5}, -\frac{53}{10} \right) \)

Ans. (2)

Sol. \( y = x^2 \), (2.4)

Tangent at (2.4) is

\( \frac{1}{2} (y - 4) = 2x \)

\( y + 4 = 4x \Rightarrow 4x - y - 4 = 0 \)

Equation of circle \( (x - 2)^2 + (y - 4)^2 + \lambda (4x - y - 4) = 0 \)

It passes through (0,1)

\( \Rightarrow 4 + 9 + \lambda (0 - 1 - 4) = 0 \)

\( 13 = 5\lambda \Rightarrow \lambda = \frac{13}{5} \)
6. If \( A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \) and \( \beta = A^4 + A \) then determinant of \( \beta \) = ?

**Ans.** (4)

**Sol.**
\[
A^4 = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} \cos^4 \theta - \sin^4 \theta & \sin \theta \cos \theta (\cos^2 \theta + \sin^2 \theta) \\ -\sin \theta \cos \theta (\cos^2 \theta + \sin^2 \theta) & \sin^4 \theta - \cos^4 \theta \end{bmatrix}
\]
\[
\beta = A^4 + A = \begin{bmatrix} \cos^4 \theta + \sin^4 \theta & \sin \theta \cos \theta (\cos^2 \theta + \sin^2 \theta) \\ -\sin \theta \cos \theta (\cos^2 \theta + \sin^2 \theta) & \sin^4 \theta + \cos^4 \theta \end{bmatrix}
\]
\[
\text{det}(\beta) = (\cos^4 \theta + \sin^4 \theta)^2 - (\sin \theta \cos \theta (\cos^2 \theta + \sin^2 \theta))^2 = 2 + 2 \cos(4\theta)
\]

7. Contrapositive of "If \( n^3 - 1 \) is even then \( n \) is odd" is

**Ans.** (3)

**Sol.**

\( P: n^3 - 1 \) is even, \( q: n \) is odd

Contrapositive of \( p \rightarrow q \) is \( \neg q \rightarrow \neg p \)

\( \neg q \rightarrow \neg p \) if \( n \) is not odd then \( n^3 - 1 \) is not even
8. If \(a_1, a_2, a_3, \ldots, a_n\) and \(b_1, b_2, b_3, \ldots\) are two arithmetic progressions with common difference of 2nd is two more than that of first and \(b_{100} = a_{100} = -399, a_{50} = -159\) then the value of \(b_1\) is

(1) \(-51\)  
(2) \(-61\)  
(3) \(-81\)  
(4) \(81\)

Ans.  
(3)

Sol.
Let \(a_1, a_2, a_3, \ldots\) first A.P.
\[
a_1 = a_1 + 39d = -159 \quad \text{--------(1)}
\]
\[
b_{100} = a_1 + 99d = -399 \quad \text{--------(2)}
\]
from equation (1) and (2)
\[
d = -4, \ a_1 = -3
\]
now
\[
b_{100} = 81
\]
\[
\Rightarrow b_1 + 99d = a_1 + 69d
\]
\[
\Rightarrow b_1 = -81 \quad \text{(According to question D = d + 2)}
\]
\[
9.
\]
If the angle of elevation of the top of a summit is 45° and a person climbs at an inclination of 30° up to 1 km, where the angle of elevation of top becomes 60°, then height of the summit is

(1) \(\frac{1}{\sqrt{3} - 1}\) km  
(2) \(\frac{\sqrt{3} - 1}{2}\) km  
(3) \(\frac{\sqrt{3} + 1}{2}\) km  
(4) \(\frac{3}{\sqrt{3} - 1}\) km

Ans.  
(3)

Sol.
\[
\sin 30° = \frac{z}{1} \quad \text{(CD = 1 km (given))}
\]
\[
z = \frac{1}{2} \quad \text{--------- (1)}
\]
\[
\cos 30° = \frac{y}{1} \Rightarrow \frac{\sqrt{3}}{2}
\]
now in \(\triangle ABC\)
\[
\tan 45° = \frac{h}{x - y}
\]
\[
\Rightarrow h = x + y
\]
\[
\Rightarrow x = h - \frac{\sqrt{3}}{2} \quad \text{--------- (2)}
\]
now in \(\triangle BDE\).

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9. \(\tan 60° = \frac{h - \frac{1}{2}}{x}\)
\[
\Rightarrow \sqrt{3} \left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2}
\]
\[
\Rightarrow (\sqrt{3} - 1)h = 1
\]
\[
h = \frac{1}{\sqrt{3} - 1} \text{ km}
\]

10. If \(C_0, C_1, C_2, \ldots, C_n\) are frequencies of \(n + 1\) observations 1, 2, 2^2, \ldots, 2^n such that mean is \(\frac{729}{2}\) then value of \(n\) is.
Sol.

<table>
<thead>
<tr>
<th>x (observation)</th>
<th>1</th>
<th>2</th>
<th>2^2</th>
<th>2^n</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (frequency)</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum f x}{\sum f}
\]

\[
1 \times 2^0 + 2 \times 2^1 + 2^2 \times 2^2 + \cdots + 2^n \times 2^n = \frac{3^3}{3!} \cdot \frac{729}{2^n}
\]

\[n = 6\]

11. Area bounded by curves \(y = x^2 - 1\) and \(y = 1 - x^2\) is

\[
\begin{align*}
(1) \quad & \frac{1}{3} \\
(2) \quad & \frac{8}{3} \\
(3) \quad & \frac{4}{3} \\
(4) \quad & \frac{2}{3}
\end{align*}
\]

Ans.

Sol.

Given curves are \(y = x^2 - 1\) and \(y = 1 - x^2\) so intersection point are \((\pm 1, 0)\)

Bounded area = \(4 \int_{-1}^{1} (1 - x^2) \, dx = 4 \left[ x - \frac{x^3}{3} \right]_{-1}^{1} = 4 \left[ 1 - \frac{1}{3} \right] = \frac{8}{3}\) sq. units
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