

FINAL JEE-MAIN EXAMINATION – SEPTEMBER, 2020
(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

1. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series

$$(x+y) + (x^2+xy+y^2) + (x^3+x^2y + xy^2+y^3)+\dots$$

(1) $\frac{x+y-xy}{(1-x)(1-y)}$ (2) $\frac{x+y-xy}{(1+x)(1+y)}$

(3) $\frac{x+y+xy}{(1+x)(1+y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$

Official Ans. by NTA (1)

2. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in

the binomial expansion of $(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}})^{10}$ is $10k$,

then k is equal to :

(1) 176 (2) 336

(3) 352 (4) 84

Official Ans. by NTA (2)

3. If a function $f(x)$ defined by

$$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \leq x < 1 \\ cx^2, & 1 \leq x \leq 3 \\ ax^2 + 2cx, & 3 < x \leq 4 \end{cases}$$

be continuous for some $a, b, c \in \mathbb{R}$ and $f'(0) + f'(2) = e$, then the value of a is :

(1) $\frac{e}{e^2 - 3e - 13}$ (2) $\frac{e}{e^2 + 3e + 13}$

(3) $\frac{1}{e^2 - 3e + 13}$ (4) $\frac{e}{e^2 - 3e + 13}$

Official Ans. by NTA (4)

TEST PAPER WITH ANSWER

4. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :

(1) $\frac{8}{17}$ (2) $\frac{2}{3}$

(3) $\frac{4}{17}$ (4) $\frac{2}{5}$

Official Ans. by NTA (1)

5. Area (in sq. units) of the region outside

$$\frac{|x|}{2} + \frac{|y|}{3} = 1 \text{ and inside the ellipse } \frac{x^2}{4} + \frac{y^2}{9} = 1$$

is :

(1) $3(4 - \pi)$ (2) $6(\pi - 2)$

(3) $3(\pi - 2)$ (4) $6(4 - \pi)$

Official Ans. by NTA (2)

6. Let S be the set of all $\lambda \in \mathbb{R}$ for which the system of linear equations

$$2x - y + 2z = 2$$

$$x - 2y + \lambda z = -4$$

$$x + \lambda y + z = 4$$

has no solution. Then the set S

(1) contains more than two elements.

(2) is a singleton.

(3) contains exactly two elements.

(4) is an empty set.

Official Ans. by NTA (3)

7. Let A be a 2×2 real matrix with entries from $\{0, 1\}$ and $|A| \neq 0$. Consider the following two statements :

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $\text{tr}(A) = 2$,

where I_2 denotes 2×2 identity matrix and $\text{tr}(A)$ denotes the sum of the diagonal entries of A . Then:

- (1) (P) is true and (Q) is false
 (2) Both (P) and (Q) are false
 (3) Both (P) and (Q) are true
 (4) (P) is false and (Q) is true

Official Ans. by NTA (4)

8. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :

- (1) If I will catch the train, then I reach the station in time.
 (2) If I do not reach the station in time, then I will not catch the train.
 (3) If I will not catch the train, then I do not reach the station in time.
 (4) If I do not reach the station in time, then I will catch the train.

Official Ans. by NTA (3)

9. Let $y = y(x)$ be the solution of the differential equation,

$$\frac{2 + \sin x}{y+1} \cdot \frac{dy}{dx} = -\cos x, y > 0, y(0) = 1. \text{ If } y(\pi) = a$$

and $\frac{dy}{dx}$ at $x = \pi$ is b , then the ordered pair

(a, b) is equal to :

- (1) (2, 1) (2) $\left(2, \frac{3}{2}\right)$
 (3) (1, -1) (4) (1, 1)

Official Ans. by NTA (4)

10. Let $X = \{x \in \mathbb{N} : 1 \leq x \leq 17\}$ and $Y = \{ax + b : x \in X \text{ and } a, b \in \mathbb{R}, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then $a + b$ is equal to :

- (1) -7 (2) 7
 (3) 9 (4) -27

Official Ans. by NTA (1)

11. If the tangent to the curve $y = x + \sin y$ at a point

(a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and

$\left(\frac{1}{2}, 2\right)$, then :

- (1) $b = a$ (2) $b = \frac{\pi}{2} + a$
 (3) $|b - a| = 1$ (4) $|a+b| = 1$

Official Ans. by NTA (3)

12. Let $P(h, k)$ be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is :

- (1) $x + 3y - 62 = 0$ (2) $x - 3y - 11 = 0$
 (3) $x - 3y + 22 = 0$ (4) $x + 3y + 26 = 0$

Official Ans. by NTA (4)

13. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, $2x = 3y, z = 1$ also passes through the point :

- (1) (0, 6, -2) (2) (-2, 0, 1)
 (3) (0, -6, 2) (4) (2, 0, -1)

Official Ans. by NTA (2)

14. Let α and β be the roots of the equation $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n, n = 1, 2, 3, \dots$, then :

- (1) $5S_6 + 6S_5 = 2S_4$
 (2) $5S_6 + 6S_5 + 2S_4 = 0$
 (3) $6S_6 + 5S_5 + 2S_4 = 0$
 (4) $6S_6 + 5S_5 = 2S_4$

Official Ans. by NTA (1)

15. If $R = \{(x,y) : x,y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers \mathbb{Z} , then the domain of R^{-1} is :

- (1) $\{-2, -1, 1, 2\}$ (2) $\{-1, 0, 1\}$
 (3) $\{-2, -1, 0, 1, 2\}$ (4) $\{0, 1\}$

Official Ans. by NTA (2)

16. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

- (1) $[-3, \infty)$ (2) $(-\infty, 9]$
 (3) $(-\infty, -9] \cup [3, \infty)$ (4) $(-\infty, -3] \cup [9, \infty)$

Official Ans. by NTA (4)

17. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1) . Then $x_1^2 + 5y_1^2$ is equal to :

- (1) 5 (2) 6
 (3) 8 (4) 10

Official Ans. by NTA (2)

18. The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to :

- (1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$
 (3) $\frac{\sqrt{17}}{2}+1$ (4) $\frac{\sqrt{17}}{2}$

Official Ans. by NTA (1)

19. The value of $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}}\right)^3$ is :

- (1) $\frac{1}{2}(\sqrt{3}-i)$ (2) $-\frac{1}{2}(\sqrt{3}-i)$
 (3) $-\frac{1}{2}(1-i\sqrt{3})$ (4) $\frac{1}{2}(1-i\sqrt{3})$

Official Ans. by NTA (2)

20. If $p(x)$ be a polynomial of degree three that has a local maximum value 8 at $x = 1$ and a local minimum value 4 at $x = 2$; then $p(0)$ is equal to:

- (1) 12 (2) -24
 (3) 6 (4) -12

Official Ans. by NTA (4)

21. The integral $\int_0^2 |x-1| - |x| dx$ is equal to _____.

Official Ans. by NTA (1.50)

22. Let \vec{a}, \vec{b} and \vec{c} be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$.

Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to _____.

Official Ans. by NTA (2.00)

23. If $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} = 820, (n \in \mathbb{N})$ then the value of n is equal to _____.

Official Ans. by NTA (40.00)

24. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is _____.

Official Ans. by NTA (309.00)

25. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is _____.

Official Ans. by NTA (9.00)