FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS — TEST PAPER WITH ANSWER

1. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$

(3) $192\sqrt{3}$ (4) $128\sqrt{3}$

Official Ans. by NTA (3)

2. Let n > 2 be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :-

(1) 199 (2) 101

(3) 201 (4) 200

Official Ans. by NTA (3)

3. If the equation $\cos^4\theta+\sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval :

(1) $\left[-\frac{3}{2},-\frac{5}{4}\right]$ (2) $\left(-\frac{1}{2},-\frac{1}{4}\right]$

(3) $\left(-\frac{5}{4},-1\right)$ (4) $\left[-1,-\frac{1}{2}\right]$

Official Ans. by NTA (4)

4. Let f(x) be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in :

(1) (–3, –1) (2) (1, 3)

(3) (–1, 0) (4) (0, 1)

Official Ans. by NTA (3)

5. Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y) \forall x,y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which $g(n) = 20$, is :

(1) 5 (2) 9

(3) 20 (4) 4

Official Ans. by NTA (1)

6. Let $a, b, c \in R$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^TA = I$, then a value of abc can be :

(1) $\frac{2}{3}$ (2) $-\frac{1}{3}$

(3) 3 (4) $\frac{1}{3}$

Official Ans. by NTA (4)

7. Let $f : (-1, \infty) \to R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1+x), x \neq 0$. Then the function f:

(1) decreases in (–1, ∞)

(2) decreases in (–1, 0) and increases in (0, ∞)

(3) increases in (–1, ∞)

(4) increases in (–1, 0) and decreases in (0, ∞)

Official Ans. by NTA (1)

8. If the sum of first 11 terms of an A.P., a_1 a_2, a_3,... is 0 ($a_1 \neq 0$), then the sum of the A.P., a_1, a_3, a_5,...,a_{23} is ka_1, where k is equal to :

(1) $\frac{121}{10}$ (2) $-\frac{72}{5}$

(3) $\frac{72}{5}$ (4) $-\frac{121}{10}$

Official Ans. by NTA (2)

9. The imaginary part of

$$\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2} \text{ can be :}$$

(1) $-2\sqrt{6}$ (2) 6

(3) $\sqrt{6}$ (4) $-\sqrt{6}$

Official Ans. by NTA (1)

10. $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to :

(1) 2 (2) e

(3) 1 (4) e^2

Official Ans. by NTA (4)

11. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :

(1) $y = 4x + 2$ (2) $x + 4y = 8$

(3) $y + 4x = 2$ (4) $2y + x = 4$

Official Ans. by NTA (2)

12. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is:

(1) $\sqrt{30}$ (2) $\frac{4\sqrt{5}}{3}$

(3) $2\sqrt{6}$ (4) $\frac{2\sqrt{5}}{3}$

Official Ans. by NTA (2)

13. Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \to q$ (2) $(q \to p) \vee \sim(p \to q)$

(3) $(p \to q) \wedge (q \to p)$ (4) $(\sim q) \vee (p \wedge q) \to q$

Official Ans. by NTA (1)

14. A plane passing through the point (3, 1,1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to:

(1) –10 (2) 5

(3) 10 (4) –5

Official Ans. by NTA (2)

15. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$.

Then $P(E_2^C \cap E_3^C / E_1)$ is equal to :

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$

(3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

Official Ans. by NTA (1)

16. Let $A = \{X = (x, y, z)^T : PX = 0$ and

$$x^2 + y^2 + z^2 = 1\} \text{ where } P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix},$$

then the set A :

(1) is a singleton

(2) contains exactly two elements

(3) contains more than two elements

(4) is an empty set

Official Ans. by NTA (2)

17. Consider a region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?

(1) $\alpha^3 - 6\alpha^2 + 16 = 0$ (2) $3\alpha^2 - 8\alpha + 8 = 0$

(3) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$ (4) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

Official Ans. by NTA (4)

18. If a curve $y = f(x)$, passing through the point $(1,2)$, is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$

(3) $\frac{-1}{1+\log_e 2}$ (4) $1+\log_e 2$

Official Ans. by NTA (2)

19. Let S be the sum of the first 9 terms of the series: $\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3+(k+4)a\} + \{x^4+(k+6)a\}+.....$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10}-x+45a(x-1)}{x-1}$, then k is equal to :

(1) –5 (2) 1

(3) –3 (4) 3

Official Ans. by NTA (3)

20. The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is :

(1) $\left(0,\frac{\pi}{4}\right)$ (2) $\left(0,\frac{3\pi}{4}\right)$

(3) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$ (4) $\left(0,\frac{\pi}{2}\right)$

Official Ans. by NTA (4)

21. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3,....b_{11}$ is 90, then the common difference of this A.P. is_______.

Official Ans. by NTA (3.00)

22. If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at $x = 0$ is________.

Official Ans. by NTA (91)

23. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$ $(\lambda > 0)$. If O is the origin and $\overrightarrow{OB}\cdot\overrightarrow{OP} - 3|\overrightarrow{OA}\times\overrightarrow{OP}|^2 = 6$, then λ is equal to______.

Official Ans. by NTA (0.8)

24. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to________.

Official Ans. by NTA (118)

25. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|dx$ is_______.

Official Ans. by NTA (1.0)