FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is :

(1) 9 (2) 18

(3) 10 (4) 20

Official Ans. by NTA (1)

2. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to :

(1) $2\sqrt{3}-\pi$ (2) $3\sqrt{2}+\pi$

(3) $3\sqrt{2}-\pi$ (4) $2\sqrt{3}+\pi$

Official Ans. by NTA (1)

3. Let R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers. Then:

(1) R_2 is transitive but R_1 is not transitive

(2) R_1 is transitive but R_2 is not transitive

(3) R_1 and R_2 are both transitive

(4) Neither R_1 nor R_2 is transitive

Official Ans. by NTA (4)

4. Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

(1) 8 (2) $4\sqrt{5}$

(3) 12 (4) $8\sqrt{5}$

Official Ans. by NTA (1)

5. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(1) $\left(x-1, \sqrt{x}\right)$ (2) $\left(x+1, \sqrt{x}\right)$

(3) $\left(x+1, -\sqrt{x}\right)$ (4) $\left(x-1, -\sqrt{x}\right)$

Official Ans. by NTA (3)

6. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(1) $\frac{121}{10^4}$ (2) $\frac{150}{10^4}$

(3) $\frac{135}{10^4}$ (4) $\frac{134}{10^4}$

Official Ans. by NTA (3)

7. If a ΔABC has vertices A(–1, 7), B(–7, 1) and C(5, –5), then its orthocentre has coordinates:

(1) (3, –3) (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

(3) (–3, 3) (4) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

Official Ans. by NTA (3)

8. If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 - 1|$, $Re(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $Im(z_1 + z_2)$ is equal to:

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $2\sqrt{3}$

Official Ans. by NTA (4)

9. The plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point :

(1) (4, 0, –1) (2) (4, 0, 1)

(3) (0, 1, –1) (4) (0, –1, 1)

Official Ans. by NTA (1)

10. $\lim\limits_{x\to a}\dfrac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}(a\neq 0)$ is equal to :

(1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

Official Ans. by NTA (1)

11. Let A be a 3 × 3 matrix such that

adj A = $\begin{bmatrix}2 & -1 & 1\\ -1 & 0 & 2\\ 1 & -2 & -1\end{bmatrix}$ and

B = adj (adj A).

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

(1) $\left(9, \frac{1}{9}\right)$ (2) $\left(9, \frac{1}{81}\right)$

(3) $\left(3, \frac{1}{81}\right)$ (4) (3, 81)

Official Ans. by NTA (3)

12. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R : f(x) = f(0)\}$, then the sum of squares of all the elements of T is :

(1) 6 (2) 8

(3) 4 (4) 2

Official Ans. by NTA (3)

13. Let a, b, c ∈ R be such that $a^2 + b^2 + c^2 = 1$. If $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is :

(1) $\frac{\pi}{2}$ (2) 0

(3) $\frac{\pi}{9}$ (4) $\frac{2\pi}{3}$

Official Ans. by NTA (1)

14. If the sum of the series

$20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+....$ upto n^{th} term is 488 and the n^{th} term is negative, then :

(1) n^{th} term is $-4\frac{2}{5}$ (2) n = 41

(3) n^{th} term is –4 (4) n = 60

Official Ans. by NTA (3)

15. Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum\limits_{i=1}^{10}(x_i - p) = 3$ and $\sum\limits_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \neq p \in R$, then the standard deviation of these observations is :

(1) $\sqrt{\frac{3}{5}}$ (2) $\frac{7}{10}$

(3) $\frac{9}{10}$ (4) $\frac{4}{5}$

Official Ans. by NTA (3)

16. If $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to :

(1) $\frac{3}{2}+\sqrt{e}$ (2) $\frac{3}{2}\sqrt{e}$

(3) $\frac{1}{2}+\sqrt{e}$ (4) $\frac{\sqrt{e}}{2}$

Official Ans. by NTA (2)

17. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1(b<5)$ and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :

(1) (8, 10) (2) (8, 12)

(3) $\left(\frac{20}{3}, 12\right)$ (4) $\left(\frac{24}{5}, 10\right)$

Official Ans. by NTA (1)

18. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(1) (–3, –1) (2) (1, 3]

(3) (0, 2) (4) (2, 4]

Official Ans. by NTA (2)

19. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then 18 k is equal to :

(1) 9 (2) 11

(3) 5 (4) 7

Official Ans. by NTA (4)

20. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) T, F, T (2) F, T, F

(3) T, T, F (4) T, T, T

Official Ans. by NTA (3)

21. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to _____.

Official Ans. by NTA (39)

22. If the tangent of the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is _____.

Official Ans. by NTA (4)

23. Let a plane P contain two lines

$\vec{r}=\hat{i}+\lambda\left(\hat{i}+\hat{j}\right)$, $\lambda \in R$ and

$\vec{r}=-\hat{j}+\mu\left(\hat{j}-\hat{k}\right)$, $\mu \in R$

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M(1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals _____.

Official Ans. by NTA (5)

24. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to _____.

Official Ans. by NTA (8)

25. The total number of 3-digit numbers, whose sum of digits is 10, is _____.

Official Ans. by NTA (54)