

FINAL JEE-MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and

$A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one

of the following is not true?

(1) $0 \leq a^2 + b^2 \leq 1$ (2) $a^2 - d^2 = 0$

(3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

Official Ans. by NTA (3)

2. Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x , $[x]^2 + 2[x + 2] - 7 = 0$ has :

(1) no integral solution

(2) exactly four integral solutions

(3) exactly two solutions

(4) infinitely many solutions

Official Ans. by NTA (4)

3. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is :

(1) 3 : 1 (2) 33 : 31

(3) 9 : 7 (4) 5 : 3

Official Ans. by NTA (3)

4. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to :

(1) 126 (2) 135

(3) 145 (4) 116

Official Ans. by NTA (1)

5. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90^\circ$, and $\text{ar}(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

(1) $2 + \sqrt{5}$ (2) $1 + \sqrt{5}$

(3) $1 + 2\sqrt{5}$ (4) $2\sqrt{5} - 1$

Official Ans. by NTA (3)

6. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$.

Then $\int_0^3 (g(x) - f(x)) dx$ is equal to :

(1) $\frac{3}{2}$ (2) 0

(3) $\frac{1}{2}$ (4) 1

Official Ans. by NTA (4)

7. Given the following two statements :

(S₁) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

(S₂) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.

Then :

(1) only (S₁) is correct.

(2) both (S₁) and (S₂) are correct.

(3) both (S₁) and (S₂) are not correct.

(4) only (S₂) is correct.

Official Ans. by NTA (3)

8. Let P(3, 3) be a point on the hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \text{ If the normal to it at P intersects}$$

the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a², e²) is equal to :

(1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{9}{2}, 2\right)$

(3) $\left(\frac{3}{2}, 2\right)$ (4) (9, 3)

Official Ans. by NTA (1)

9. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$ (x ≥ 0). Then f(3) – f(1)

is equal to :

(1) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Official Ans. by NTA (4)

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

(1) 65 (2) 37

(3) 29 (4) 55

Official Ans. by NTA (4)

11. Let $u = \frac{2z+i}{z-ki}$, z = x + iy and k > 0. If the curve

represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where PQ = 5, then the value of k is :

(1) 3/2 (2) 4

(3) 2 (4) 1/2

Official Ans. by NTA (3)

12. Let x₀ be the point of local maxima of

$$f(x) = \vec{a} \cdot (\vec{b} \times \vec{c}), \text{ where } \vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k},$$

$$\vec{b} = -2\hat{i} + x\hat{j} - \hat{k} \text{ and } \vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}. \text{ Then the}$$

value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at x = x₀ is :

(1) -30 (2) 14

(3) -4 (4) -22

Official Ans. by NTA (4)

13. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

(1) 20/3 (2) 5

(3) 10/3 (4) 6

Official Ans. by NTA (4)

14. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(1) 7 (2) 3

(3) 5 (4) 9

Official Ans. by NTA (1)

15. The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to :

(where C is a constant of integration)

(1) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$

(2) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$

(3) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

(4) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$

Official Ans. by NTA (4)

16. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+\dots+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to :

- (1) (10, 97) (2) (11, 103)
(3) (10, 103) (4) (11, 97)

Official Ans. by NTA (2)

17. Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x \cos x + \sin x)$, $x > 0$.

If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to :

- (1) $2 + \frac{\pi}{2}$ (2) $1 + \frac{\pi}{2}$
(3) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (4) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

Official Ans. by NTA (1)

18. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

- (1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$
(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Official Ans. by NTA (2)

19. Let f be a twice differentiable function on $(1, 6)$. If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then :

- (1) $f(5) \leq 10$ (2) $f'(5) + f''(5) \leq 20$
(3) $f(5) + f'(5) \geq 28$ (4) $f(5) + f'(5) \leq 26$

Official Ans. by NTA (3)

20. If $(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$,

where $a > b > 0$, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is :

- (1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$
(3) $\frac{2a+b}{2a-b}$ (4) $\frac{a-2b}{a+2b}$

Official Ans. by NTA (2)

21. If the system of equations

$$x - 2y + 3z = 9$$

$$2x + y + z = b$$

$$x - 7y + az = 24,$$

has infinitely many solutions, then $a - b$ is equal to _____ .

Official Ans. by NTA (5)

22. The probability of a man hitting a target is $\frac{1}{10}$.

The least number of shots required, so that the probability of his hitting the target at least once

is greater than $\frac{1}{4}$, is _____.

Official Ans. by NTA (3)

23. Suppose a differentiable function $f(x)$ satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$,

for all real x and y . If $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$, then $f'(3)$

is equal to _____.

Official Ans. by NTA (10)

24. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is

equal to _____.

Official Ans. by NTA (8)

25. If the equation of a plane P , passing through the intesection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in \mathbb{R}$, then the distance of the point $(3, 2, -1)$ from the plane P is _____.

Official Ans. by NTA (3)