

## FINAL JEE-MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04<sup>th</sup> SEPTEMBER, 2020) TIME : 3 PM to 6 PM

### MATHEMATICS

### TEST PAPER WITH ANSWER

1. The function  $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \leq 1 \\ \frac{1}{2}(|x|-1), & |x| > 1 \end{cases}$  is :

- (1) continuous on  $\mathbb{R} - \{1\}$  and differentiable on  $\mathbb{R} - \{-1, 1\}$ .
- (2) both continuous and differentiable on  $\mathbb{R} - \{-1\}$ .
- (3) continuous on  $\mathbb{R} - \{-1\}$  and differentiable on  $\mathbb{R} - \{-1, 1\}$ .
- (4) both continuous and differentiable on  $\mathbb{R} - \{1\}$

**Official Ans. by NTA (1)**

2. Let  $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i = T$ , where each  $X_i$  contains 10

elements and each  $Y_i$  contains 5 elements. If each element of the set  $T$  is an element of exactly 20 of sets  $X_i$ 's and exactly 6 of sets  $Y_i$ 's, then  $n$  is equal to :

- (1) 45
- (2) 15
- (3) 50
- (4) 30

**Official Ans. by NTA (4)**

3. Let  $\lambda \neq 0$  be in  $\mathbb{R}$ . If  $\alpha$  and  $\beta$  are the roots of the equation,  $x^2 - x + 2\lambda = 0$  and  $\alpha$  and  $\gamma$  are the roots of the equation,  $3x^2 - 10x + 27\lambda = 0$ ,

then  $\frac{\beta\gamma}{\lambda}$  is equal to :

- (1) 36
- (2) 27
- (3) 9
- (4) 18

**Official Ans. by NTA (4)**

4. The solution of the differential equation

$$\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0 \text{ is :-}$$

(where  $C$  is a constant of integration.)

- (1)  $x - 2 \log_e(y+3x) = C$
- (2)  $x - \log_e(y+3x) = C$

(3)  $x - \frac{1}{2} (\log_e(y+3x))^2 = C$

(4)  $y + 3x - \frac{1}{2} (\log_e x)^2 = C$

**Official Ans. by NTA (3)**

5. Let  $a_1, a_2, \dots, a_n$  be a given A.P. whose common difference is an integer and  $S_n = a_1 + a_2 + \dots + a_n$ . If  $a_1 = 1, a_n = 300$  and  $15 \leq n \leq 50$ , then the ordered pair  $(S_{n-4}, a_{n-4})$  is equal to :

- (1) (2480, 249)
- (2) (2490, 249)
- (3) (2490, 248)
- (4) (2480, 248)

**Official Ans. by NTA (3)**

6. The distance of the point  $(1, -2, 3)$  from the plane  $x - y + z = 5$  measured parallel to the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \text{ is :}$$

- (1) 7
- (2) 1
- (3)  $\frac{1}{7}$
- (4)  $\frac{7}{5}$

**Official Ans. by NTA (2)**

7. Let  $f : (0, \infty) \rightarrow (0, \infty)$  be a differentiable function such that  $f(1) = e$  and

$$\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$$

If  $f(x) = 1$ , then  $x$  is equal to :

- (1)  $2e$
- (2)  $\frac{1}{2e}$
- (3)  $e$
- (4)  $\frac{1}{e}$

**Official Ans. by NTA (4)**

8. If the system of equations  
 $x + y + z = 2$   
 $2x + 4y - z = 6$   
 $3x + 2y + \lambda z = \mu$   
 has infinitely many solutions, then :  
 (1)  $\lambda - 2\mu = -5$                       (2)  $2\lambda - \mu = 5$   
 (3)  $2\lambda + \mu = 14$                       (4)  $\lambda + 2\mu = 14$   
**Official Ans. by NTA (3)**
9. The minimum value of  $2^{\sin x} + 2^{\cos x}$  is :-  
 (1)  $2^{1-\frac{1}{\sqrt{2}}}$                                       (2)  $2^{-1+\sqrt{2}}$   
 (3)  $2^{1-\sqrt{2}}$                                       (4)  $2^{-1+\frac{1}{\sqrt{2}}}$   
**Official Ans. by NTA (1)**
10.  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3 x \cdot \sin^2 3x (2\sec^2 x \cdot \sin^2 3x + 3 \tan x \cdot \sin 6x) dx$   
 is equal to :  
 (1)  $\frac{9}{2}$                       (2)  $-\frac{1}{9}$                       (3)  $-\frac{1}{18}$                       (4)  $\frac{7}{18}$   
**Official Ans. by NTA (3)**
11. The circle passing through the intersection of the circles,  $x^2 + y^2 - 6x = 0$  and  $x^2 + y^2 - 4y = 0$ , having its centre on the line,  $2x - 3y + 12 = 0$ , also passes through the point :  
 (1) (1, -3)                                      (2) (-1, 3)  
 (3) (-3, 1)                                      (4) (-3, 6)  
**Official Ans. by NTA (4)**
12. The angle of elevation of a cloud C from a point P, 200 m above a still lake is  $30^\circ$ . If the angle of depression of the image of C in the lake from the point P is  $60^\circ$ , then PC (in m) is equal to :  
 (1) 400                                      (2)  $400\sqrt{3}$   
 (3) 100                                      (4)  $200\sqrt{3}$   
**Official Ans. by NTA (1)**
13. If a and b are real numbers such that  
 $(2 + \alpha)^4 = a + b\alpha$ , where  $\alpha = \frac{-1+i\sqrt{3}}{2}$ , then  
 a + b is equal to :  
 (1) 57                      (2) 33                      (3) 24                      (4) 9  
**Official Ans. by NTA (4)**

14. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six The game stops as soon as either of the players wins. The probability of A winning the game is :  
 (1)  $\frac{31}{61}$                       (2)  $\frac{5}{6}$                       (3)  $\frac{5}{31}$                       (4)  $\frac{30}{61}$   
**Official Ans. by NTA (4)**
15. Let  $x = 4$  be a directrix to an ellipse whose centre is at the origin and its eccentricity is  $\frac{1}{2}$ .  
 If P (1,  $\beta$ ),  $\beta > 0$  is a point on this ellipse, then the equation of the normal to it at P is :-  
 (1)  $7x - 4y = 1$                                       (2)  $4x - 2y = 1$   
 (3)  $4x - 3y = 2$                                       (4)  $8x - 2y = 5$   
**Official Ans. by NTA (2)**
16. Contrapositive of the statement:  
 'If a function f is differentiable at a, then it is also continuous at a', is :-  
 (1) If a function f is continuous at a, then it is not differentiable at a.  
 (2) If a function f is not continuous at a, then it is differentiable at a.  
 (3) If a function f is not continuous at a, then it is not differentiable at a.  
 (4) If a function f is continuous at a, then it is differentiable at a.  
**Official Ans. by NTA (3)**
17. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola,  $y = x^2 - 1$  below the x-axis, is :  
 (1)  $\frac{4}{3\sqrt{3}}$                       (2)  $\frac{1}{3\sqrt{3}}$                       (3)  $\frac{4}{3}$                       (4)  $\frac{2}{3\sqrt{3}}$   
**Official Ans. by NTA (1)**

18. If for some positive integer  $n$ , the coefficients of three consecutive terms in the binomial expansion of  $(1+x)^{n+5}$  are in the ratio  $5 : 10 : 14$ , then the largest coefficient in this expansion is :-

- (1) 792      (2) 252      (3) 462      (4) 330

**Official Ans. by NTA (3)**

19. If the perpendicular bisector of the line segment joining the points  $P(1, 4)$  and  $Q(k, 3)$  has  $y$ -intercept equal to  $-4$ , then a value of  $k$  is :-

- (1)  $\sqrt{15}$       (2)  $-2$       (3)  $\sqrt{14}$       (4)  $-4$

**Official Ans. by NTA (4)**

20. Suppose the vectors  $x_1, x_2$  and  $x_3$  are the solutions of the system of linear equations,  $Ax = b$  when the vector  $b$  on the right side is equal to  $b_1, b_2$  and  $b_3$  respectively. If

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of}$$

$A$  is equal to :-

- (1)  $\frac{1}{2}$       (2) 4      (3)  $\frac{3}{2}$       (4) 2

**Official Ans. by NTA (4)**

21. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is \_\_\_\_\_

**Official Ans. by NTA (135)**

22. Let  $PQ$  be a diameter of the circle  $x^2+y^2=9$ . If  $\alpha$  and  $\beta$  are the lengths of the perpendiculars from  $P$  and  $Q$  on the straight line,  $x + y = 2$  respectively, then the maximum value of  $\alpha\beta$  is \_\_\_\_\_

**Official Ans. by NTA (7)**

23. Let  $\{x\}$  and  $[x]$  denote the fractional part of  $x$  and the greatest integer  $\leq x$  respectively of a

real number  $x$ . If  $\int_0^n \{x\} dx, \int_0^n [x] dx$  and  $10(n^2 - n)$ , ( $n \in \mathbb{N}, n > 1$ ) are three consecutive terms of a G.P., then  $n$  is equal to \_\_\_\_\_

**Official Ans. by NTA (21)**

24. If  $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ , then the value of

$|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$  is equal to \_\_\_\_\_

**Official Ans. by NTA (18)**

25. If the variance of the following frequency distribution :

Class	: 10–20	20–30	30–40
Frequency	: 2	x	2

is 50, then  $x$  is equal to \_\_\_\_\_

**Official Ans. by NTA (4)**