

FINAL JEE-MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Saturday 05th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. If $3^{2 \sin 2\alpha} - 1$, 14 and $3^{4 - 2 \sin 2\alpha}$ are the first three terms of an A.P. for some α , then the sixth term of this A.P. is :

- (1) 66 (2) 65
(3) 81 (4) 78

Official Ans. by NTA (1)

2. If the function $f(x) = \begin{cases} k_1(x - \pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$

is twice differentiable, then the ordered pair (k_1, k_2) is equal to :

- (1) $\left(\frac{1}{2}, 1\right)$ (2) (1, 1)
(3) $\left(\frac{1}{2}, -1\right)$ (4) (1, 0)

Official Ans. by NTA (1)

3. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to :

- (1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$
(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4}$

Official Ans. by NTA (3)

4. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

- (1) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$
(2) $(x \wedge \sim y) \vee (\sim x \wedge y)$
(3) $(x \wedge y) \vee (\sim x \wedge \sim y)$
(4) $(x \wedge y) \wedge (\sim x \vee \sim y)$

Official Ans. by NTA (3)

5. If the volume of a parallelepiped, whose coterminal edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and

$\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu. units, then :

- (1) $\vec{a} \cdot \vec{c} = 17$ (2) $\vec{b} \cdot \vec{c} = 10$
(3) $n = 7$ (4) $n = 9$

Official Ans. by NTA (2)

6. If $y = y(x)$ is the solution of the differential

equation $\frac{5 + e^x}{2 + y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying

$y(0) = 1$, then a value of $y(\log_e 13)$ is :

- (1) 1 (2) -1
(3) 2 (4) 0

Official Ans. by NTA (2)

7. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

- (1) 63 (2) 38
(3) 54 (4) 36

Official Ans. by NTA (4)

8. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is

- (1) $\frac{25}{9}$ (2) $\frac{25}{81}$
(3) $\frac{5}{27}$ (4) $\frac{5}{9}$

Official Ans. by NTA (2)

9. If $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x+e^{-x})} dx$
 = $g(x)e^{(e^x+e^{-x})} + c$, where c is a constant of
 integration, then $g(0)$ is equal to :

- (1) 2 (2) e^2
 (3) e (4) 1

Official Ans. by NTA (1)

10. If the minimum and the maximum values of the
 function $f : \left[\frac{\pi}{4}, \frac{\pi}{2} \right] \rightarrow \mathbb{R}$, defined by :

$$f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair
 (m, M) is equal to :

- (1) $(0, 4)$ (2) $(-4, 4)$
 (3) $(0, 2\sqrt{2})$ (4) $(-4, 0)$

Official Ans. by NTA (4)

11. Let $\lambda \in \mathbb{R}$. The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

is inconsistent for :

- (1) exactly one negative value of λ .
 (2) exactly one positive value of λ .
 (3) every value of λ .
 (4) exactly two values of λ .

Official Ans. by NTA (1)

12. If S is the sum of the first 10 terms of the series

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \dots,$$

then $\tan(S)$ is equal to :

- (1) $\frac{5}{11}$ (2) $-\frac{6}{5}$
 (3) $\frac{10}{11}$ (4) $\frac{5}{6}$

Official Ans. by NTA (4)

13. If the four complex numbers $z, \bar{z}, \bar{z} - 2\text{Re}(\bar{z})$
 and $z - 2\text{Re}(z)$ represent the vertices of a square
 of side 4 units in the Argand plane, then $|z|$ is
 equal to :

- (1) 4 (2) 2
 (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Official Ans. by NTA (4)

14. If the point P on the curve, $4x^2 + 5y^2 = 20$ is
 farthest from the point $Q(0, -4)$, then PQ^2 is
 equal to :

- (1) 21 (2) 36
 (3) 48 (4) 29

Official Ans. by NTA (2)

15. The mean and variance of 7 observations are
 8 and 16, respectively. If five observations are
 2, 4, 10, 12, 14, then the absolute difference
 of the remaining two observations is :

- (1) 2 (2) 4
 (3) 3 (4) 1

Official Ans. by NTA (1)

16. If (a, b, c) is the image of the point $(1, 2, -3)$
 in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then
 $a + b + c$ is equal to

- (1) -1 (2) 2
 (3) 3 (4) 1

Official Ans. by NTA (2)

17. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ is

(1) π (2) $\frac{3\pi}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Official Ans. by NTA (4)

18. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to :

(1) $\frac{3^{11}}{2} + 2^{10}$ (2) $3^{11} - 2^{12}$

(3) 3^{11} (4) $2 \cdot 3^{11}$

Official Ans. by NTA (3)

19. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to :

(1) 8 (2) 6

(3) 16 (4) 9

Official Ans. by NTA (1)

20. If α is the positive root of the equation,

$$p(x) = x^2 - x - 2 = 0, \text{ then } \lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4}$$

is equal to

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{3}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

Official Ans. by NTA (1)

21. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is _____.

Official Ans. by NTA (11)

22. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$

and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and

$6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is _____

Official Ans. by NTA (30)

23. The natural number m, for which the coefficient

of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$

is 1540, is _____.

Official Ans. by NTA (13)

24. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _____.

Official Ans. by NTA (240)

25. Let $f(x) = x \cdot \left[\frac{x}{2}\right]$, for $-10 < x < 10$, where [t]

denotes the greatest integer function. Then the number of points of discontinuity of f is equal to _____.

Official Ans. by NTA (8)