FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Saturday 05th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to :

(1) 9 (2) –3

(3) –9 (4) 4

Official Ans. by NTA (2)

2. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to :

(1) $\frac{27}{16}$ (2) $\frac{1}{24}$

(3) $\frac{27}{32}$ (4) $\frac{3}{8}$

Official Ans. by NTA (1)

3. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \ldots$ is 460, then x is equal to :

(1) $7^{46/21}$ (2) $7^{1/2}$

(3) e^2 (4) 7^2

Official Ans. by NTA (4)

4. $\lim\limits_{x\to 0} \frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

(1) does not exist. (2) is equal to $\sqrt{e}$.

(3) is equal to 0. (4) is equal to 1.

Official Ans. by NTA (4)

5. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is :

(1) $\frac{2}{13}(3^{50}-1)$ (2) $\frac{1}{26}(3^{50}-1)$

(3) $\frac{1}{13}(3^{50}-1)$ (4) $\frac{1}{26}(3^{49}-1)$

Official Ans. by NTA (2)

6. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is :

(1) 2^{15} i (2) -2^{15}

(3) -2^{15} i (4) 6^5

Official Ans. by NTA (3)

7. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is :

(1) $\frac{\sqrt{3}}{12}$ (2) $\frac{\sqrt{3}}{10}$

(3) $\frac{2\sqrt{3}}{5}$ (4) $\frac{2\sqrt{3}}{3}$

Official Ans. by NTA (2)

8. The area (in sq. units) of the region $A = \{(x, y) : (x - 1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where [t] denotes the greatest integer function, is :

(1) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2} - 1$

(3) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (4) $\frac{4}{3}\sqrt{2} + 1$

Official Ans. by NTA (1)

9. If the length of the chord of the circle, $x^2 + y^2 = r^2$ $(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to :

(1) $\frac{9}{5}$ (2) $\frac{12}{5}$

(3) 12 (4) $\frac{24}{5}$

Official Ans. by NTA (2)

10. If $x = 1$ is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then :

(1) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

(2) $x = 1$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.

(3) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f.

(4) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f.

Official Ans. by NTA (1)

11. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

(1) $2x^2 - 20x + 19 = 0$

(2) $x^2 - 10x + 19 = 0$

(3) $x^2 - 10x + 18 = 0$

(4) $x^2 - 20x + 18 = 0$

Official Ans. by NTA (2)

12. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$$ is equal to :

(1) 0 (2) $y(a - b)$

(3) $y(b - a)$ (4) $y(a - c)$

Official Ans. by NTA (2)

13. If $\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}\,d\theta = A\log_e|B(\theta)| + C$, where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be :

(1) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$ (2) $\frac{2\sin\theta+1}{\sin\theta+3}$

(3) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$ (4) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$

Official Ans. by NTA (4)

14. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

(1) $5m = 4$ (2) $4c^2 = 369$

(3) $c^2 = 369$ (4) $8m + 5 = 0$

Official Ans. by NTA (2)

15. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is :

(1) 1500 (2) 2255

(3) 3000 (4) 2250

Official Ans. by NTA (4)

16. If for some $\alpha \in R$, the lines

$L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :

(1) (–2, 10, 2) (2) (10, 2, 2)

(3) (10, –2, –2) (4) (2, –10, –2)

Official Ans. by NTA (4)

17. Let y = y(x) be the solution of the differential equation $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to :

(1) $\sqrt{2}-2$ (2) $\frac{1}{\sqrt{2}} - 1$

(3) $2 - \sqrt{2}$ (4) $2 + \sqrt{2}$

Official Ans. by NTA (1)

18. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0) ?

(1) (2, 2) (2) (–2, 6)

(3) (–2, 4) (4) (2, 6)

Official Ans. by NTA (2)

19. The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is:

(1) a contradiction

(2) equivalent to $(p \wedge q) \vee (\sim q)$

(3) a tautology

(4) equivalent to $(p \vee q) \wedge (\sim p)$

Official Ans. by NTA (3)

20. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then :

(1) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(2) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

(3) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

(4) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

Official Ans. by NTA (1)

21. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _____.

Official Ans. by NTA (11.00)

22. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is _____.

Official Ans. by NTA (19.00)

23. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is _____.

Official Ans. by NTA (120.00)

24. If the lines x + y = a and x – y = b touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to_____.

Official Ans. by NTA (0.50)

25. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is _____.

Official Ans. by NTA (6.00)