FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 06th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

(1) $\left(-1,\sqrt{3}\right)$ (2) $\left(-1,\sqrt{2}\right)$

(3) $\left(-2,\sqrt{3}\right)$ (4) (1,2)

Official Ans. by NTA (1)

2. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) 2!3!4! (2) $(3!)^3.(4!)$

(3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

Official Ans. by NTA (2)

3. $$\lim_{x\to1}\left(\frac{\int_0^{(x-1)^2} t\cos\left(t^2\right)dt}{(x-1)\sin(x-1)}\right)$$

(1) does not exist (2) is equal to $\frac{1}{2}$

(3) is equal to 1 (4) is equal to $-\frac{1}{2}$

Official Ans. by NTA (1)
Official Ans. by ALLEN
(Bonus-Answers musbe zero)

4. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$

(3) $\frac{3}{8}$ (4) $\frac{7}{8}$

Official Ans. by NTA (1)

5. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively

(1) 5 and 7 (2) 6 and 8

(3) 4 and 9 (4) 5 and 8

Official Ans. by NTA (4)

6. The area (in sq. units) of the region $A = \{(x,y) : |x| + |y| \le 1,\ 2y^2 \ge |x|\}$ is :

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$

(3) $\frac{7}{6}$ (4) $\frac{5}{6}$

Official Ans. by NTA (4)

7. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

(1) $\frac{15}{101}$ (2) $\frac{5}{101}$

(3) $\frac{5}{33}$ (4) $\frac{10}{99}$

Official Ans. by NTA (3)

8. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n,a > 1)$ then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

(1) $n\sqrt{a-1}$ (2) $\sqrt{a-1}$

(3) $a - 1$ (4) $\sqrt{n(a-1)}$

Official Ans. by NTA (2)

9. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) $x + 3 = 0$ (2) $x + 2y = 0$

(3) $2x + 1 = 0$ (4) $x + 2 = 0$

Official Ans. by NTA (1)

10. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

Official Ans. by NTA (3)

11. If $f(x + y) = f(x)\,f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Official Ans. by NTA (2)

12. The general solution of the differential equation

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0 \text{ is :}$$

(where C is a constant of integration)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

Official Ans. by NTA (1)

13. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line x=l at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (2) $(3, -\sqrt{3})$

(3) $\left(4, -\frac{\sqrt{3}}{2}\right)$ (4) $(4, -\sqrt{3})$

Official Ans. by NTA (2)

14. Let a,b,c,d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then :

(1) a,c,p are in G.P. (2) a,c,p are in A.P.

(3) a,b,c,d are in G.P. (4) a,b,c,d are in A.P.

Official Ans. by NTA (3)

15. If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$

(3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

Official Ans. by NTA (1)

16. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1,t_2]$ is attained at the point :

(1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$

(3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

Official Ans. by NTA (4)

17. The region represented by $\{z = x + iy \in C : |z| - Re(z) \le 1\}$ is also given by the inequality :

(1) $y^2 \ge x + 1$ (2) $y^2 \ge 2(x + 1)$

(3) $y^2 \le x + \frac{1}{2}$ (4) $y^2 \le 2\left(x+\frac{1}{2}\right)$

Official Ans. by NTA (4)

18. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

(1) 1 (2) 3

(3) 4 (4) 2

Official Ans. by NTA (4)

19. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is :

(1) $\frac{1}{2}$ (2) 1

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans. by NTA (4)

20. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}.$$ Then the ordered pair (m,M) is equal to

(1) (–3,–1) (2) (–4,–1)

(3) (1,3) (4) (–3,3)

Official Ans. by NTA (1)

21. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is_.

Official Ans. by NTA (5.00)

22. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is_.

Official Ans. by NTA (80.00)

23. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _.

Official Ans. by NTA (28.00)

24. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\left|\vec{a}+\vec{b}\right|+\left|\vec{a}-\vec{b}\right|$ is _.

Official Ans. by NTA (4.00)

25. Let $f : R \to R$ be defined as

$$f(x)=\begin{cases} x^5\sin\left(\frac{1}{x}\right)+5x^2 & , \quad x<0 \\ 0 & , \quad x=0 \\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2 & , \quad x>0 \end{cases}$$

. The value of λ for which $f''(0)$ exists, is _.

Official Ans. by NTA (5.00)