

FINAL JEE-MAIN EXAMINATION – SEPTEMBER, 2020
(Held On Wednesday 06th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS

1. The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x)(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is :

(1) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$
(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

Official Ans. by NTA (4)

2. For all twice differentiable functions $f : \mathbb{R} \rightarrow \mathbb{R}$, with $f(0) = f(1) = f'(0) = 0$
- (1) $f''(x) = 0$, for some $x \in (0, 1)$
(2) $f''(0) = 0$
(3) $f''(x) \neq 0$ at every point $x \in (0, 1)$
(4) $f''(x) = 0$ at every point $x \in (0, 1)$

Official Ans. by NTA (1)

3. If the tangent to the curve, $y = f(x) = x \log_e x$, ($x > 0$) at a point $(c, f(c))$ is parallel to the line - segment joining the points $(1, 0)$ and (e, e) , then c is equal to :

(1) $\frac{1}{e-1}$ (2) $e^{\left(\frac{1}{1-e}\right)}$
(3) $e^{\left(\frac{1}{e-1}\right)}$ (4) $\frac{e-1}{e}$

Official Ans. by NTA (3)

TEST PAPER WITH ANSWER

4. Consider the statement : "For an integer n , if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is :

- (1) For an integer n , if $n^3 - 1$ is not even, then n is not odd.
(2) For an integer n , if n is even, then $n^3 - 1$ is odd.
(3) For an integer n , if n is odd, then $n^3 - 1$ is even.
(4) For an integer n , if n is even, then $n^3 - 1$ is even.

Official Ans. by NTA (2)

5. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

- (1) $e^2 + 2e - 1 = 0$ (2) $e^2 + e - 1 = 0$
(3) $e^4 + 2e^2 - 1 = 0$ (4) $e^4 + e^2 - 1 = 0$

Official Ans. by NTA (4)

6. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be $(1, 1, 2)$. Then the equation of the line through this centroid and perpendicular to the plane P is :

(1) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$
(2) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$
(3) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$
(4) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

Official Ans. by NTA (2)

7. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :

- (1) $2\alpha^2$ (2) $2\alpha(\alpha + 1)$
 (3) $-2\alpha(\alpha + 1)$ (4) $2\alpha(\alpha - 1)$

Official Ans. by NTA (3)

8. Let $z = x + iy$ be a non-zero complex number such that $z^2 = iz|z|^2$, where $i = \sqrt{-1}$, then z lies on the :

- (1) imaginary axis (2) real axis
 (3) line, $y = x$ (4) line, $y = -x$

Official Ans. by NTA (3)

9. The common difference of the A.P. b_1, b_2, \dots, b_m is 2 more than the common difference of A.P. a_1, a_2, \dots, a_n . If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to :

- (1) -127 (2) -81
 (3) 81 (4) 127

Official Ans. by NTA (2)

10. The angle of elevation of the summit of a mountain from a point on the ground is 45° . After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60° . Then the height (in km) of the summit from the ground is :

- (1) $\frac{1}{\sqrt{3}-1}$ (2) $\frac{1}{\sqrt{3}+1}$
 (3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (4) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Official Ans. by NTA (1)

11. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$. If $B = A + A^4$, then $\det(B)$:

- (1) is one (2) lies in (1, 2)
 (3) is zero (4) lies in (2, 3)

Official Ans. by NTA (2)

12. For a suitably chosen real constant a , let a function, $f : \mathbb{R} - \{-a\} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{a-x}{a+x}. \text{ Further suppose that for any real}$$

number $x \neq -a$ and $f(x) \neq -a$, $(f \circ f)(x) = x$. Then

$f\left(-\frac{1}{2}\right)$ is equal to :

- (1) $\frac{1}{3}$ (2) 3
 (3) -3 (4) $-\frac{1}{3}$

Official Ans. by NTA (2)

13. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in \mathbb{R} , where f is not differentiable. Then :

- (1) $\{0, 1\}$ (2) $\{0\}$
 (3) \emptyset (an empty set) (4) $\{1\}$

Official Ans. by NTA (1)

14. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :

- (1) $\frac{4}{3}$ (2) $\frac{8}{3}$
 (3) $\frac{16}{3}$ (4) $\frac{7}{2}$

Official Ans. by NTA (2)

15. The probabilities of three events A, B and C are given by $P(A) = 0.6, P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8, P(A \cap C) = 0.3, P(A \cap B \cap C) = 0.2, P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval:

- (1) $[0.36, 0.40]$ (2) $[0.35, 0.36]$
 (3) $[0.25, 0.35]$ (4) $[0.20, 0.25]$

Official Ans. by NTA (3)

16. if the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals :

- (1) 2 (2) 1
(3) 3 (4) 9

Official Ans. by NTA (3)

17. The integral $\int_1^2 e^x \cdot x^x (2 + \log_e x) dx$ equal :

- (1) $e(4e + 1)$ (2) $e(2e - 1)$
(3) $4e^2 - 1$ (4) $e(4e - 1)$

Official Ans. by NTA (4)

18. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point $(-1, -4)$ in this line is :

- (1) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (2) $\left(\frac{29}{5}, \frac{11}{5}\right)$
(3) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (4) $\left(\frac{29}{5}, \frac{8}{5}\right)$

Official Ans. by NTA (3)

19. If $y = \left(\frac{2}{\pi}x - 1\right) \operatorname{cosec} x$ is the solution of the differential equation,

$\frac{dy}{dx} + p(x)y = \frac{2}{\pi} \operatorname{cosec} x$, $0 < x < \frac{\pi}{2}$, then the function $p(x)$ is equal to

- (1) $\cot x$ (2) $\tan x$
(3) $\operatorname{cosec} x$ (4) $\sec x$

Official Ans. by NTA (1)

20. The centre of the circle passing through the point $(0, 1)$ and touching the parabola $y = x^2$ at the point $(2, 4)$ is :

- (1) $\left(\frac{3}{10}, \frac{16}{5}\right)$ (2) $\left(\frac{-16}{5}, \frac{53}{10}\right)$
(3) $\left(\frac{6}{5}, \frac{53}{10}\right)$ (4) $\left(\frac{-53}{10}, \frac{16}{5}\right)$

Official Ans. by NTA (2)

21. The sum of distinct values of λ for which the system of equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$$

has non-zero solutions, is _____.

Official Ans. by NTA (3.00)

22. Suppose that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(1) = 3$.

If $\sum_{i=1}^n f(i) = 363$, then n is equal to

_____.

Official Ans. by NTA (5.00)

23. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____.

Official Ans. by NTA (120.00)

24. Consider the data on x taking the values $0, 2, 4, 8, \dots, 2^n$ with frequencies ${}^n C_0, {}^n C_1, {}^n C_2, \dots, {}^n C_n$ respectively. If the mean of this data is

$$\frac{728}{2^n},$$

then n is equal to _____.

Official Ans. by NTA (6.00)

25. If \vec{x} and \vec{y} be two non-zero vectors such that

$$|\vec{x} + \vec{y}| = |\vec{x}| \text{ and } 2\vec{x} + \lambda\vec{y} \text{ is perpendicular to}$$

\vec{y} , then the value of λ is _____.

Official Ans. by NTA (1.00)